# Concrete basic creep and Poisson's ratios: back to basics of viscoelasticity and applications

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## Résumé

*Des matériaux comme le béton ou les polymères présentent du fluage. Sous chargement uniaxial maintenu constant, des déformations différées à la fois axiales et transverses sont observées. La généralisation du coefficient de Poisson au contexte du fluage n'étant pas univoque, de nombreuses définitions différentes ont été proposées dans la littérature associée au béton. De plus, l'évolution attendue en fonction du temps (croissante, décroissante, non monotone) des coefficients de Poisson viscoélastiques a fait l'objet de débats. Cette contribution propose de revenir aux bases de la viscoélasticité linéaire, afin de rappeler les définitions des coefficients de Poisson de fluage et de relaxation. L'équivalent viscoélastique du classique formulaire reliant les caractéristiques élastiques isotropes est aussi rappelé sous une forme synthétique. Les possibles évolutions des coefficients de Poisson sont illustrées sur plusieurs exemples académiques et pratiques, sur le béton et les polymères. Les résultats sont issus à la fois de modèles et de données expérimentales.*

## Abstract

*Materials such as concrete and polymers exhibit creep. Under sustained and constant uniaxial stress loading, axial and transverse delayed strains are observed. Generalization of the elastic Poisson's ratio to the context of creep being ambiguous, numerous different definitions have been proposed in the literature associated to concrete. Moreover, the expected evolution with respect to time (increasing, decreasing, non monotonic) of viscoelastic Poisson's ratios has been a subject of debate. This contribution proposes to go back to basics of linear viscoelasticity, to recall the definitions of creep and relaxation Poisson's ratios. The viscoelastic equivalents of the classical relations between the elastic isotropic characteristics are also recalled in a compact form. The possible evolutions of viscoelastic Poisson's ratios are illustrated on various examples, both academic and practical, on concrete and polymers. Results come from both models and experimental data.*

**Keywords: concrete, viscoelasticity, Poisson's ratio, modelling, experimental**

# 1 Introduction

Materials such as concrete and polymers exhibit creep. Under sustained and constant uniaxial stress loading, axial and transverse delayed strains are observed. The scientific community has thus adapted the isotropic elastic characteristics, Young's modulus and Poisson's ratio, to the creep context, introducing time dependencies. While the definition of the uniaxial creep compliance (generalizing the inverse of the Young's modulus) is unambiguous, it is not the case regarding generalization of the Poisson's ratio. Indeed, in the literature dedicated to concrete, especially in the experimental domain, numerous different definitions of delayed Poisson's ratios have been proposed. Moreover, the expected time evolution has been a subject of debate, notably regarding its monotonic (or not) character. More details and an extensive bibliographic review can be found in [1].

On the other hand, theory of isotropic linear viscoelasticity has allowed, for several decades, to establish rigorous generalizations of elastic Young's modulus and Poisson's ratio.

This communication first proposes to recall the definition of creep and relaxation Poisson's ratios, and their relations to the bulk and shear relaxation or compliance functions. Expressions are provided in the ageing context as it is more general than the non ageing one; restrictions to the latter are straightforward [1]. Second, possible evolutions of both Poisson's ratios are illustrated on various examples. Even if the presented elements, especially in the first part, are not new, gathering them in this communication seems useful to help to share knowledge between the mechanics and concrete material science communities, to improve how the multiaxial behaviour of concrete is studied.

Both academic and practical examples (on concrete and polymers) from models and experimental tests are provided. Their aim is to show that a great diversity of time evolutions can be encountered for both the creep and relaxation Poisson's ratios: increasing, decreasing, non monotonic, one being monotonic while the other is not.

# 2 Material behaviour in ageing linear viscoelasticity

This section introduces the ageing linear viscoelastic behaviour, in the general anisotropic case, then particularized to isotropy. The uniaxial creep and relaxation experiments are modelled, to derive the creep and relaxation Poisson's ratios (respectively denoted CPR and RPR in the following). Eventually, useful relations between viscoelastic material characteristics are gathered.

Even if these developments can be found in classical textbooks, they introduce the notations adopted in this paper, and are recalled for comprehensiveness. Only the ageing case is presented, the equivalent relations in the non ageing case can be derived replacing functions of time by their Laplace-Carson transform and Volterra products by classical products in the Laplace-Carson domain (see [1] for more details).

## 2.1 Anisotropic linear viscoelasticity

The linear viscoelastic behaviour is completely defined by either the relaxation or the compliance tensor [2]. The component $ijkl$ of the relaxation tensor $\mathbb{R}(t, t')$ is equal to the stress along $i, j$ arising at time $t$ for a unit strain loading step along $k, l$ occuring at $t'$. Conversely, the component $ijkl$ of the compliance tensor $\mathbb{J}(t, t')$ is equal to the strain along $i, j$ arising at time $t$ for a unit stress loading step along $k, l$ occuring at $t'$. Both tensors are equal to 0 when $t < t'$, due to the causality principle.

The mechanical behaviour is then written for any strain or stress history, taking advantage of linearity (Boltzmann principle):

$$\boldsymbol{\varepsilon}(t) = \int_{t'=-\infty}^{t} \mathbb{J}(t,t') : \mathrm{d}\boldsymbol{\sigma}(t') \quad \text{or} \quad \boldsymbol{\varepsilon}(t) = \mathbb{J}(t,.) \,\vdots\, \boldsymbol{\sigma}(.) \quad \text{or} \quad \boldsymbol{\varepsilon} = \mathbb{J} \,\vdots\, \boldsymbol{\sigma} \tag{1}$$

$$\boldsymbol{\sigma}(t) = \int_{t'=-\infty}^{t} \mathbb{R}(t,t') : \mathrm{d}\boldsymbol{\varepsilon}(t') \quad \text{or} \quad \boldsymbol{\sigma}(t) = \mathbb{R}(t,.) \,\vdots\, \boldsymbol{\varepsilon}(.) \quad \text{or} \quad \boldsymbol{\sigma} = \mathbb{R} \,\vdots\, \boldsymbol{\varepsilon} \tag{2}$$

using Stieltjes integrals [3, 4], and where " $\vdots$ " denotes the so-called Volterra integral tensor operator, in memory of Volterra's pioneering works [3]. The relaxation and compliance tensors are inverses in the sense of the Volterra integral tensor operator (H being the Heaviside function):

$$\int_{t'=-\infty}^{t} \mathbb{R}(t,t^0) : \mathrm{d}\mathbb{J}(t^0,t') = \mathrm{H}(t-t')\mathbb{I} \quad \text{or} \quad \mathbb{R}(t,.) \,\vdots\, \mathbb{J}(.,t') = \mathrm{H}(t-t')\mathbb{I} \quad \text{or} \quad \mathbb{R} \,\vdots\, \mathbb{J} = \mathrm{H}\mathbb{I} \tag{3}$$

## 2.2 Isotropic linear viscoelasticity

A presentation of this topic was given by Mandel as early as 1958 [5]. The relations presented here are close to Mandel ones except that analogs of the bulk and shear moduli in elasticity are used instead of Lamé coefficients.

### 2.2.1 General stress or strain histories

The material being isotropic, the behaviour can now be described by two scalar functions. A convenient way to write the isotropic behaviour is to express the relaxation and compliance tensors on the basis $\mathbb{J}, \mathbb{K}$ of isotropic fourth order tensors:

$$\mathbb{R}(t,t') = 3R_k(t,t')\mathbb{J} + 2R_g(t,t')\mathbb{K} \quad \text{and} \quad \mathbb{J}(t,t') = \frac{1}{3}J_k(t,t')\mathbb{J} + \frac{1}{2}J_g(t,t')\mathbb{K} \tag{4}$$

introducing the bulk $R_k$ and shear $R_g$ relaxation functions, and the bulk $J_k$ and shear $J_g$ compliance functions. The scalar factors 3 and 2 are introduced to mimic the expressions of the elastic isotropic stiffness and compliance tensors as functions of bulk and shear moduli.

According to (3), the bulk (resp. shear) relaxation and compliance functions are inverses in the sense of the Volterra integral operator:

$$R_k \circ J_k = \mathrm{H} \quad \text{and} \quad R_g \circ J_g = \mathrm{H} \tag{5}$$

The aim is now to derive the ageing linear viscoelastic "equivalents" of the Young's modulus and Poisson's ratio, by analogy with their definition in elasticity. This is performed simulating uniaxial creep and relaxation experiments, and using the recalled ageing linear viscoelastic behaviour to compute strain or stress evolutions.

### 2.2.2 Case of the uniaxial creep experiment

In the uniaxial creep experiment, for any loading time $t_0$, the stress tensor evolution is prescribed as:

$$\boldsymbol{\sigma}(t) = \sigma_{11}^0 \mathrm{H}(t - t_0)\underline{e}_1 \otimes \underline{e}_1 \tag{6}$$

The "uniaxial compliance function" $J_E$ is then defined as:

$$J_E(t, t_0) = \frac{\varepsilon_{11}(t)}{\sigma_{11}(t)} = (J_k/9 + J_g/3)(t, t_0) \tag{7}$$

While the "creep Poisson's ratio" $\nu_c$ is defined as:

$$\nu_c(t, t_0) = -\frac{\varepsilon_{22}(t)}{\varepsilon_{11}(t)} = -\frac{(J_k/9 - J_g/6)(t, t_0)}{(J_k/9 + J_g/3)(t, t_0)} \tag{8}$$

As is well-known, the CPR is not constant unless the bulk and shear compliances are proportional which might be reasonable or not depending on the considered material. These equations (7, 8) are identical to equations given by Bažant [6, 7].

The inverses of the compliances (but not directly the relaxations) and the CPR verify the same relations as their counterparts (in terms of inverses of stiffnesses) in elasticity. Let us also note that the partial time derivative of the CPR can be conveniently written as a function of the ratio of the shear and bulk compliances:

$$\frac{\partial \nu_c}{\partial t}(t, t_0) = \frac{9}{2} \frac{\frac{\partial (J_g/J_k)}{\partial t}(t, t_0)}{\left(3\frac{J_g}{J_k}(t, t_0) + 1\right)^2} \tag{9}$$

This relation has the important consequence that the time variations of the CPR are identical to those of the ratio of the shear and bulk compliances.

### 2.2.3 Case of the uniaxial relaxation experiment

In the uniaxial relaxation experiment, the evolution of the component 11 of the strain is prescribed as:

$$\varepsilon_{11}(t) = \varepsilon_{11}^0 \mathrm{H}(t - t_0) \tag{10}$$

while the components 22, 33, 23, 13, 12 of the stress tensor are constantly equal to 0. The "uniaxial relaxation function" $R_E$ is then defined as:

$$\frac{1}{R_E(t, t_0)} = \frac{\varepsilon_{11}(t)}{\sigma_{11}(t)} = \frac{1}{(J_k/9 + J_g/3)^{-1}(t, t_0)} \tag{11}$$

where the inverse "$^{-1}$" is defined in the sense of the Volterra integral operator "$\circ$". While the "relaxation Poisson's ratio" $\nu_r$ is defined as:

$$\nu_r(t, t_0) = -\frac{\varepsilon_{22}(t)}{\varepsilon_{11}(t)} = -\left((J_k/9 - J_g/6) \circ (J_k/9 + J_g/3)^{-1}\right)(t, t_0) \tag{12}$$

In the framework of non ageing linear viscoelasticity, this coefficient has been defined in the Laplace-Carson domain by [4, 2] (under the name relaxation Poisson's ratio) and [8] (under the name viscoelastic

Poisson's ratio), and also in the general case and in the time domain by [9].

The RPR can also be written:

$$\nu_r(t, t_0) = \left((2R_k + 2R_g/3)^{-1} \circ (R_k - 2R_g/3)\right)(t, t_0) \tag{13}$$

This relation is similar to a relation given by Mandel [5].

#### 2.2.4 Relations between viscoelastic properties

The main relations between compliance and relaxation functions, and viscoelastic Poisson's ratios are gathered in table 1. The fact that the uniaxial compliance and relaxation functions are inverses (in the sense of the Volterra integral operator) comes from (7) and (11). The relation between both Poisson's ratios is derived combining (8), (12) and (7). Note that the product between $\nu_c$ and $J_E$ is the usual product between scalars, while the product between $\nu_r$ and $J_E$ is the Volterra integral operator. This relation can be found, for example, in [2, 9]. As mentioned by [10], it is different from the relation derived by Lakes and Wineman in [11].

| | | | |
|---|---|---|---|
| $J_E = (J_k + 3J_g)/9$ | | $R_E = 9R_g \circ (R_g + 3R_k)^{-1} \circ R_k$ | |
| $\nu_c = (3J_g - 2J_k)/\left[2(3J_g + J_k)\right]$ | | $\nu_r = \left[2(3R_k + R_g)\right]^{-1} \circ (3R_k - 2R_g)$ | |
| $J_k = 3(1 - 2\nu_c)J_E$ | | $R_k = (R_E/3) \circ (\mathrm{H} - 2\nu_r)^{-1}$ | |
| $J_g = 2(1 + \nu_c)J_E$ | | $R_g = (R_E/2) \circ (\mathrm{H} + \nu_r)^{-1}$ | |
| $R_k \circ J_k = \mathrm{H}$ | $R_g \circ J_g = \mathrm{H}$ | $R_E \circ J_E = \mathrm{H}$ | $\nu_c J_E = \nu_r \circ J_E$ |

Table 1: A summary of useful relations in ageing linear viscoelasticity (note that the inverse "$^{-1}$" is defined in the sense of the Volterra integral product "$\circ$", while "/" is the classical scalar division).

# 3 Discussion on creep and relaxation Poisson's ratios

Even if the aforementioned theory has been known for a long time in the continuum mechanics community, discussions are still needed on some issues (see [1] for more discussions).

## 3.1 Variation of the viscoelastic Poisson's ratios

As shown in equation (9), the partial time derivative with respect to $t$ of the CPR can be related to the time derivative of the ratio $J_g/J_k$, and has the same sign. This makes clearer the common sense that if creep is faster in shear, the CPR increases, while if it is faster in volume, it decreases. In fact it is not the ratio of the derivatives of the compliances which is concerned, but the derivative of the ratio of the compliances. Therefore, if the ratio $J_g/J_k$ decreases, the CPR also decreases. However, the authors were not able to derive a similar general result on the variation of the RPR, which would have been useful to verify Tschoegl's statement [12] that the viscoelastic Poisson's ratio (corresponding to our RPR) is a non-decreasing function of time. Instead of that, practical examples will be shown in section 4 both in ageing and non ageing viscoelasticity, in contradiction with that statement which was already pointed out to be wrong by Lakes [11].

## 3.2 Correspondence principle and viscoelastic Poisson's ratio

It has been argued in the literature that the elastic/viscoelastic analogy does not apply to the viscoelastic Poisson's ratio [13]. It has been shown here that one just needs to be careful so as to correctly define the Poisson's ratio in viscoelasticity. If one deals with the CPR, which is commonly used by experimental researchers, particularly in the field of concrete science since it is straightforward to compute from experiments, the correspondence principle cannot be used. However, the latter can be used with the RPR, which can be easily computed if the CPR and the uniaxial compliance are known (the numerical inversion of the integral equation can be done following [14, 15]).

As mentioned earlier, due to the validity of the correspondence principle for the RPR, some authors have used it to define the RPR directly in the transformed domain [2, 8, 16, 11, 12, 17].

## 3.3 Comparison with Hilton's classification

It has been argued by Hilton [13], amongst others, that the viscoelastic Poisson's ratio is load-history dependent, and that this fact prevents from using these functions to describe the material behaviour in general cases. Here, following early works on linear viscoelasticity, it has been shown that although the viscoelastic Poisson's ratio is load-history dependent, it is perfectly consistent to accept this fact and define two particular cases of viscoelastic Poisson's ratio which are called relaxation and creep Poisson's ratio since they are equal to the opposite ratio of transverse to axial strains in experiments of the same name. Both these coefficients can be used in the time domain to describe generally the behaviour of any isotropic linear viscoelastic solid. However, it has also been shown that only the RPR can be used in the transformed domain in the non ageing case.

It is interesting to note that the two viscoelastic Poisson's ratio defined here are not perfectly consistent with the five classes of Poisson's ratios defined by Hilton [18, 19]. Class I corresponds to the opposite of the ratio of transverse to axial strains, without further precisions about the loading. Class II is based on the same equation, except that the axial strain is constant, without further information about strains or stresses in the other directions. Class III is based on Fourier transform but again the loading is not specified. Class IV and V are based on the Hencky strain and on the strain rates respectively. Therefore, the definitions used in this paper both belong to Class I, for two different particular loadings corresponding to the uniaxial creep and relaxation experiments. Some additional classes of viscoelastic Poisson's ratios are defined in [20], but again the distinction between CPR and RPR (both belonging to class I) is not made, and the relation between these coefficients is not given.

Finally, both Poisson's ratio are valid material parameters, each having advantages or disadvantages depending on the application. However, the use of bulk and shear compliance or relaxation functions might be preferable, as they are unambiguous and might thus be less error prone.

# 4 Applications

## 4.1 Poisson's ratios evolutions from theoretical models

The aim of these academic examples, from straightforward behaviours, is to illustrate the fact that viscoelastic Poisson's ratios can be increasing, decreasing or even non monotonic functions of time.

### 4.1.1 Non ageing viscoelasticity: Maxwell matrix and elastic inclusions

A composite material made up of elastic inclusions embedded into a Maxwell non ageing viscoelastic matrix is first considered. To focus on the influence of the mechanical interactions between inclusions and matrix, the elastic and viscous Poisson's ratios of matrix are taken as equal. The CPR and RPR of matrix are thus constant. The matrix relaxation tensor reads:

$$\mathbb{R}_m(t,t') = E_m \mathrm{e}^{-\frac{t-t'}{\tau_m}} \mathrm{H}(t-t') \left( \frac{1}{1-2\nu_m} \mathbb{J} + \frac{1}{1+\nu_m} \mathbb{K} \right) \tag{14}$$

with $E_m$ the elastic Young's modulus, $\nu_m$ the Poisson's ratio, $\tau_m$ the Maxwell characteristic time. Inclusions are elastic, characterised by the Young's modulus $E_i$ and the Poisson's ratio $\nu_i$. The volume fraction of inclusions is denoted by $f_i$. The effective behaviour of this matrix-inclusions composite material is estimated using the Mori-Tanaka scheme [21] and the non ageing correspondence principle (taking advantage of the Laplace-Carson transform). The bulk and shear effective behaviours are found to correspond to generalized Maxwell models [22], with two Maxwell chains.

Even with such a straightforward (two Maxwell chains) rheological model, various evolutions (decreasing, non-monotonic) of Poisson's ratios are possible, see figure 1 left, where both Poisson's ratios of matrix and inclusions are constant and equal. This is in contradiction with Tschoegl statement that viscous Poisson's ratio are non-decreasing [12]. It is even possible to encounter simultaneously a monotonic CPR and a non monotonic RPR, see case $f_i = 0.5$ on figure 1 right. As can be easily shown (see [10]), the initial and final values of the CPR and RPR are identical in the non ageing context.

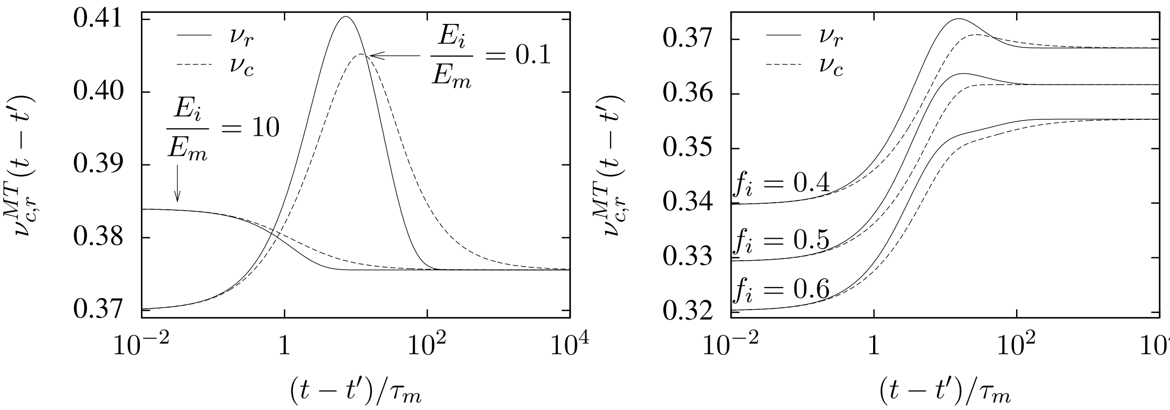


Figure 1: Non ageing Maxwell matrix and elastic inclusions: effective CPR and RPR, influence of elastic contrast between inclusions and matrix ($\nu_i = \nu_m = 0.4$, $f_i = 0.3$) and of inclusions volume fraction ($E_i/E_m = 0.1$, $\nu_i = 0.3$, $\nu_m = 0.4$).

### 4.1.2 Ageing viscoelasticity: Bažant solidification theory

To extend the non ageing analysis proposed by [10], a first application to ageing viscoelastic behaviours is considered, using Bažant solidification theory [23] to define the bulk and shear behaviours. In this theory, here extended to tensors, a non ageing relaxation tensor $\mathbb{R}^{na}$ is multiplied by a so-called ageing function $f^a$ depending on loading time $t'$:

$$\mathbb{R}(t,t') = f^a(t')\mathbb{R}^{na}(t-t') \tag{15}$$

For the sake of simplicity, the non ageing behaviour is here represented by an isotropic Maxwell model. Spherical and deviatoric viscoelastic properties (stiffness and viscosity) are assumed to be different:

$$\mathbb{R}^{na}(t-t') = \left(3k\mathrm{e}^{-\frac{t-t'}{\eta/k}}\mathbb{J} + 2g\mathrm{e}^{-\frac{t-t'}{\gamma/g}}\mathbb{K}\right)\mathrm{H}(t-t') \tag{16}$$

Alternatively, the elastic (springs stiffness) and viscous (dashpots viscosity) parts of the isotropic Maxwell behaviour can be defined introducing the elastic and viscous Young's modulus $E^e$, $E^v$ and Poisson's ratio $\nu^e, \nu^v$, from the classical relations:

$$k = \frac{E^e}{3(1-2\nu^e)}\,, \quad g = \frac{E^e}{2(1+\nu^e)}\,, \quad \eta = \frac{E^v}{3(1-2\nu^v)} \quad \text{and} \quad \gamma = \frac{E^v}{2(1+\nu^v)} \tag{17}$$

The ageing function is taken as:

$$f^a(t') = f^{a0} + (f^{a\infty} - f^{a0})\left(1 - \mathrm{e}^{-(t'/\tau^a)^2}\right) \tag{18}$$

where $f^{a0}$ and $f^{a\infty}$ are respectively the initial and final values, and $\tau^a$ is the ageing characteristic time. While both Poisson's ratios have the same initial value (at $t \to t'$) and rate, the final values (when $t \to \infty$) are found to differ (figure 2), contrary to the non ageing case. Furthermore, in this application, the CPR seems to converge when $t \to \infty$ towards a unique value irrespective of the loading time $t'$, while it is not the case for the RPR. And the latter reaches much faster its asymptotic value than the CPR.

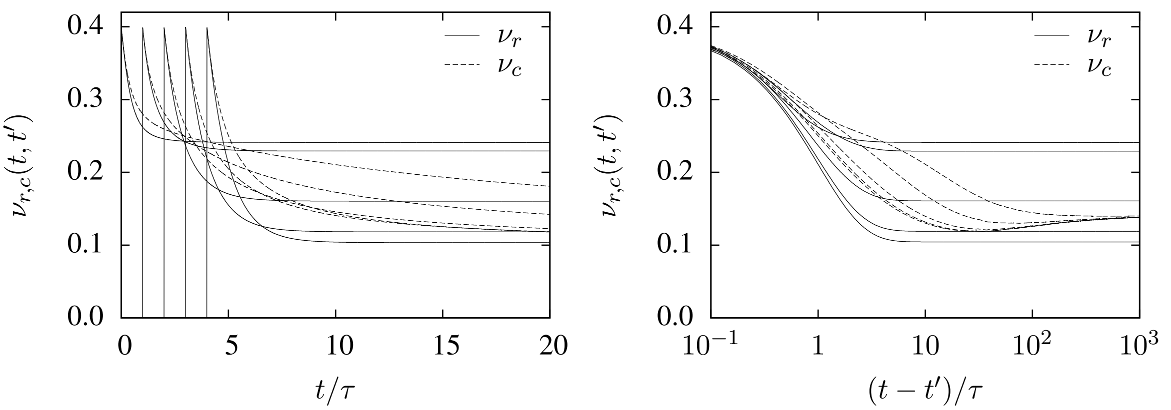


Figure 2: CPR and RPR for $t'/\tau = 0, 1, 2, 3, 4$, linear and logarithmic time scales ($\nu^e = 0.4$, $\nu^v = 0.1$, $f^{a0} = 0.1$, $f^{a\infty} = 1$, $\tau^a/\tau = 2$).

## 4.2 Concrete

Concrete being a material exhibiting creep, CPR evolutions are investigated from both models and experimental tests.

#### 4.2.1 Multiscale estimation of creep Poisson's ratio using *Vi(CA)$_2$T*

*Vi(CA)$_2$T* (Virtual Concrete And Cement Ageing Analysis Toolbox) is a software dedicated to the prediction of concrete properties, developed at EDF R&D since 2006 [24, 25]. Basically, input data is the concrete mix design, properties of cement and aggregates used, as well as the mechanical and physical properties of the hydrates. First, a hydration module computes the evolution of volume fractions of the various phases of concrete (anhydrous, water and hydrates). Second, a morphological model describes the multiscale arrangement of those phases in RVEs as well as the shapes of the inclusion phases (figure 3). Finally, micromechanical models predict elasticity (based on [26]) and basic creep on microstructures frozen once loaded, thanks to recent developments based on [27, 28]. Since the microstructure evolves before loading, the creep response depends on the loading time. Assuming microstructure as frozen once loaded, the non ageing correspondence principle can be used. Promising approaches to overcome this limitation are currently developed [29, 30].

The elementary creep mechanism is still a matter of debate in the scientific community. Even once assuming creep as originating from C-S-H "elementary bricks", various investigations on the delayed strains at this scale can be found:

- Analysing many basic creep results from the literature, [31] showed by downscaling that the elementary creep strains in or in-between elementary bricks cannot be purely deviatoric, when considering these elementary particles as spherical. Non spherical (eg. oblate) shapes would yield a different conclusion.

- Performing unjacketed (confining and pore pressures set as equal) creep tests on cement pastes, [32] measured creep strains which are negligible: the spherical creep strain of the solid skeleton of these cement pastes is negligible.

Nevertheless, the mechanism assumed here is deviatoric as in [28]: sliding at the scale of sheets in C-S-H elementary bricks.

The CPR is very high at the lowest scales (C-S-H gels), which means that creep is almost completely deviatoric (figure 4 left). This is consistent with the fact that the assumed elementary creep mechanism is deviatoric. At upper scales, due to the incorporation of porosity and stiffer elastic inclusions, a larger part of creep occurs under spherical loading. This induces a much lower CPR, which happens to remain almost constant. The fact that, as scaling up, the asymptotic CPR gets closer to $0.2$ is consistent with the asymptotic analysis performed by [31].

This practical application (see more details in [33]) shows that CPR variations can be very diverse, including non-monotonic (as for cement paste, see figure 4 left).

Apart from the C-S-H particles behaviour, the choice of the Poisson's ratio of aggregates ($\nu_{agg} = 0.27$ here) can have an impact on concrete CPR. A sensitivity analysis has been performed, increasing or decreasing aggregates Poisson's ratio by $10\%$. At the concrete scale, the elastic Poisson's ratio is modified by less than $5\%$, and the CPR is found to be less and less sensitive to the aggregates Poisson's ratio as time increases (figure 4 right).

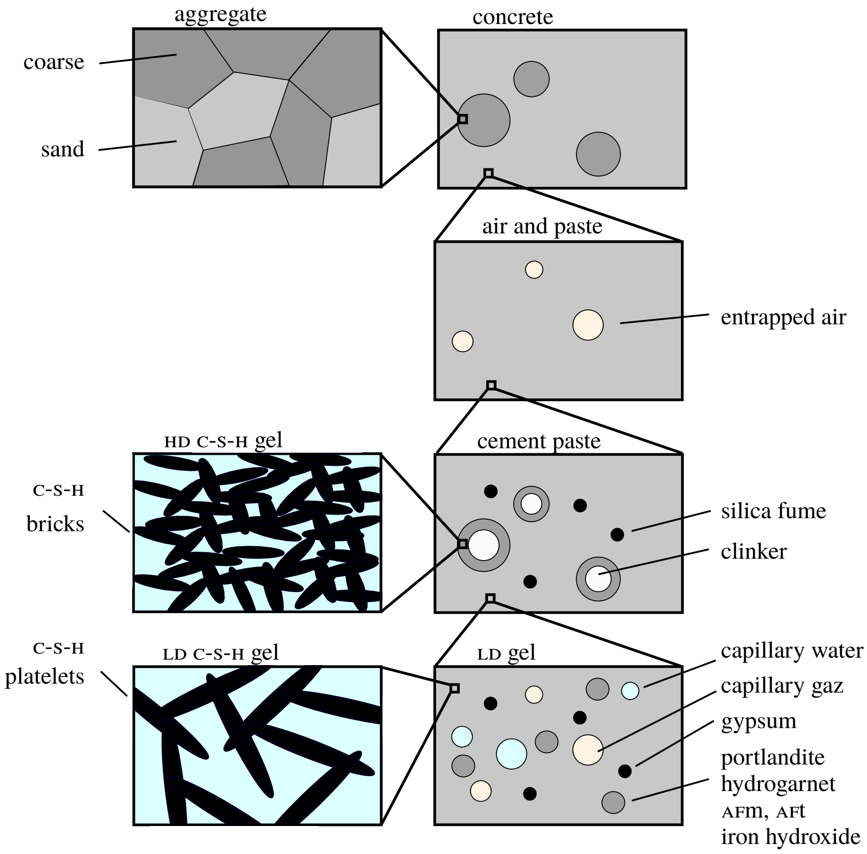


Figure 3: Multiscale morphology of concrete used in *Vi(CA)$_2$T* V2.1.2.

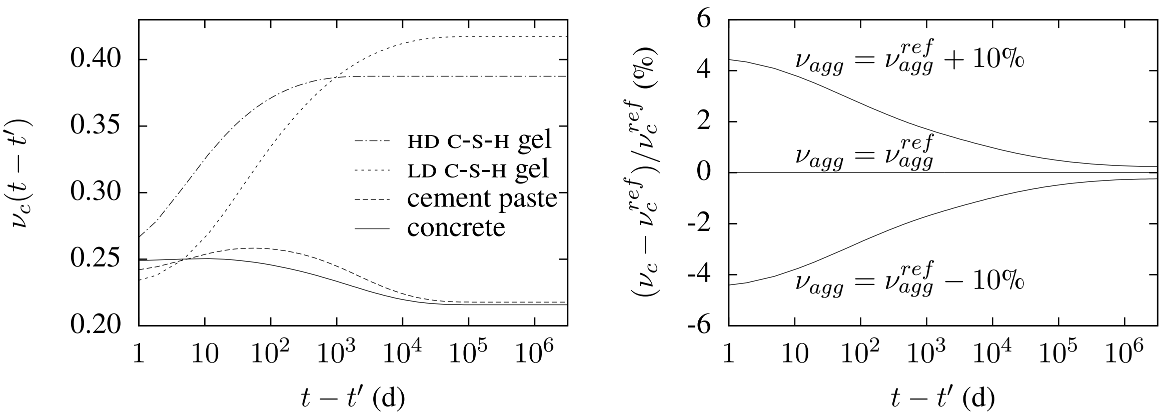


Figure 4: Left: CPRs estimated by *Vi(CA)$_2$T* V2.1.2 at the main levels of the morphological model, loading at 90 days. Right: Sensitivity of CPR of concrete estimated by *Vi(CA)$_2$T* V2.1.2, with respect to Poisson's ratio of aggregates.

### 4.2.2 Computation of creep Poisson's ratio from biaxial tests

This final application to concrete proposes an experimental illustration. Biaxial creep tests were started at EDF in 2004 in order to gain a better knowledge of the multiaxial behaviour of concrete in nuclear power plants concrete containment buildings. These tests have been described in [34] and will be the focus of a detailed paper [35].

Under a biaxial state of stress, where stresses are applied in the vertical ($h$ subscript) and horizontal ($v$) directions, leaving the third direction unloaded, a direct application of (1) and (4) yields:

$$\nu_c(t) = \frac{\sigma_h \varepsilon_v(t) - \sigma_v \varepsilon_h(t)}{\sigma_v \varepsilon_v(t) - \sigma_h \varepsilon_h(t)} \tag{19}$$

This CPR has been computed for basic creep (difference between the strains measured in the loaded non drying test and in the non loaded non drying test) and is plotted on figure 5. It is found to be almost constant. Surprisingly, it is rather high (around 0.3) compared to the aforementioned model results and to the experimental data collected by [31].

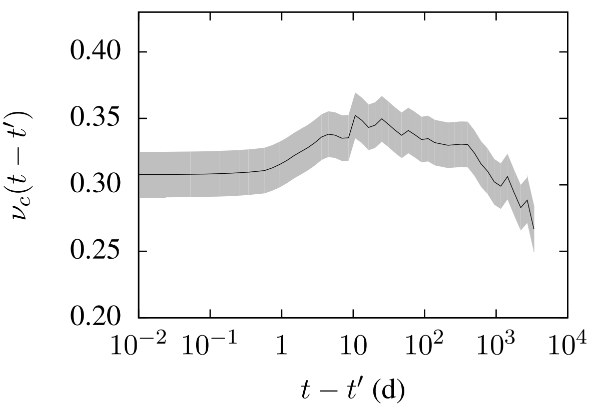


Figure 5: CPR computed from basic creep biaxial tests performed at EDF, loading at 90 days (the shaded area represents the uncertainty related to the thermal dilation of the sample due to temperature variations in the testing room).

## 4.3 Amorphous polymers: Poisson's ratios from bulk and shear behaviours

Polymers also exhibit a viscoelastic mechanical behaviour. Grassia et al. [36] have collected from the literature the bulk and shear behaviours of several amorphous polymers. They have computed the RPR considering the material as non ageing, taking advantage of the Laplace-Carson transform, from the non ageing equivalent of relation given in table 1. From this experimental evidence, the RPR is found to be slightly non monotonic for one polymer (polycarbonate).

Reusing the bulk and shear compliance or relaxation functions reported by [36], both the RPR and CPR are directly computed in the time domain, using expressions from table 1. Results are plotted on figure 6 for both polycarbonate and polycyanurate, $x_M = 0.1$ (the latter being the mole fraction of monofunctional monomer used in material preparation). The RPR obtained by [36] is plotted as dots. The RPRS are consistent, up to the fact that bulk and shear compliance or relaxation functions have been manually digitized on [36], yielding some noise especially at lower times. The CPRs are found to be slightly lower compared to the relaxation ones, as also numerically evidenced on theoretical models (subsection 4.1.1) for cases where the Poisson's ratios are increasing.

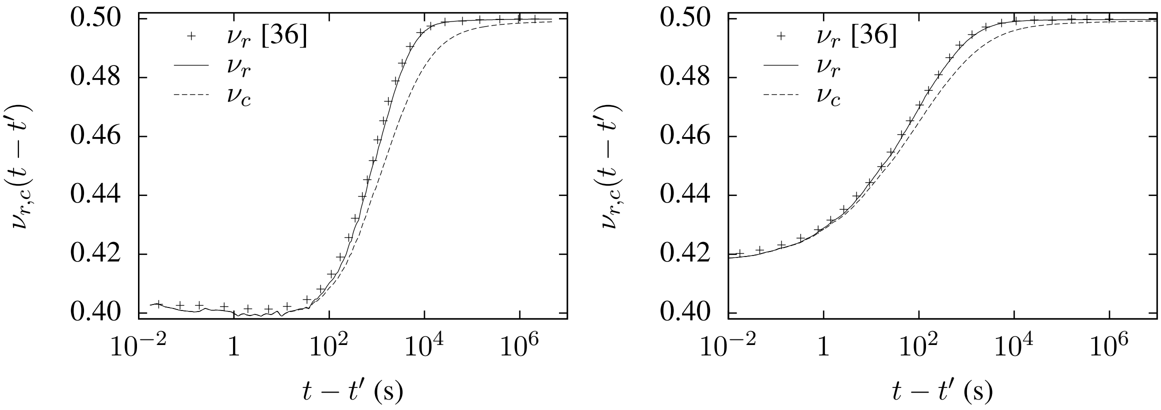


Figure 6: CPR and RPR of polycarbonate (left) and polycyanurate, $x_M = 0.1$ (right), computed from data in [36].

The advantage of the CPR computation proposed here is that it only requires the inversion of relaxation functions to compliances, which was performed by numerical time discretization. Therefore, no direct or inverse Laplace transform was needed. One can also note that the relation (9) is available on the CPR. This relation could have brought interesting information in the discussion about the variations of the viscoelastic Poisson's ratios depending on those of the compliances dealt with in [36].

# 5 Conclusion

Several definitions of viscoelastic Poisson's ratios have been introduced by various scientific communities, notably in concrete and polymer fields. This paper proposes rederivations of both the creep and relaxation Poisson's ratios using classical integral expressions of the linear viscoelastic behaviour. Practical relations between isotropic linear viscoelastic characteristics (relaxation and compliance functions, Poisson's ratios) are gathered, similarly to the classical relations between isotropic elastic characteristics. Eventually, several examples, both theoretical and practical, about concrete and polymers, show that the evolution of both Poisson's ratios can be non monotonic and quite diverse.

Restrictions on viscoelastic characteristics have been studied in a rather large extent in the non ageing case. However, up to our knowledge, the literature still lacks such comprehensive analyses in the ageing case. Inequalities can be found but they are more often based on intuition than on thermodynamics, and having particular materials in mind, such as concrete. In the same line of thought, restrictions on the viscoelastic Poisson's ratios are not clear: depending on the author, the latter can be either monotonic or non monotonic. Here, the possible non monotonicity has been shown resorting to examples. Even if the CPR can be straigthforwardly bounded by $-1$ and $1/2$ as in elasticity, these bounds are only verified on examples as far as the RPR is concerned.

Eventually, regarding both material scale modelling and structure scale computations, while in elasticity the isotropic behaviour can be indifferently characterized by either the bulk and shear moduli or the Young's modulus and Poisson's ratio; in linear viscosity using bulk and shear relaxation (or compliance) functions seems to be less error prone, as their definition is unambiguous.

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