Reliability based optimization of a hat stiffened panel

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Abstract

This article is the first of its kind that treats hat stiffened panel in reliability based optimization (RBO). This panel is used in aerospace and marine structure. Along RBO, a deterministic optimization of this panel is performed. These optimization methods are based on a polynomial metamodel. Panel optimization aims to maximize its rigidity and minimize its weight. The variables vector contains the panel geometric dimensions. Uniform distributions are associated to geometric variables and materials densities of the panel. The results of these two optimization types are compared to show the influence of inputs uncertainties on the optimum design. RBO is performed using two methods; The first being a combination of Monte-Carlo simulation to propagate uncertainty and NSGA2 to find optimum points. The second uses analytic calculation to propagate uncertainty and bi-objective Normal boundary intersection to find the Pareto front. The latter outperforms the first and produces an exact Pareto front with a better execution time.

Key words: hat stiffened panel / reliability based optimization / NBI / NSGA2 /analytic calculation.

1 Introduction

In recent years, optimization under uncertainty has widely evolved. This type of optimization is highly demanded in mechanical engineering. This demand is due to the irreducible presence of uncertainties sources like manufacturing tolerances, materials properties, temperature, humidity, etc. This optimization type incorporates two different approaches: Reliability based optimization [1] and Robust optimization (RO) [2]. The RBO results are more reliable, while those of RO are less sensible to inputs variations.

Composite panels are widely employed in manufacturing. The hat stiffened panel is used in aerospace and marine constructions. (Fig. 1) [4] shows an application of this panel in Boein 787. These applications require a high level of reliability. In this paper a RBO is applied to a hat stiffened panel that was studied in [3] with deterministic optimization. This article is

Figure 1: Application of hat stiffened panel in fuselage Boeing 787

the first work that studies RBO of a hat stiffened panel. Here, the Pareto front is computed by maximizing the panel's rigidity and minimizing its weight. The problem is formulated as follows. The variables are defined as the panel geometric dimensions. At first, a polynomial metamodel is constructed to replace the rigidity function. In a second place, a deterministic optimization and RBO are performed. RBO is realized in two methods: the first being the combination of Monte-Carlo (MC) simulations for uncertainty propagation and NSGA2 [5] as optimization algorithm. The second being the combination of analytic calculation (AC) to propagate uncertainty and Normal boundary intersection (NBI) [6] to construct the Pareto front. The inputs uncertainties are associated to the problem variables and materials densities of the panel. The results of these methods are compared and the advantages of each method are identified.

2 Panel and case study

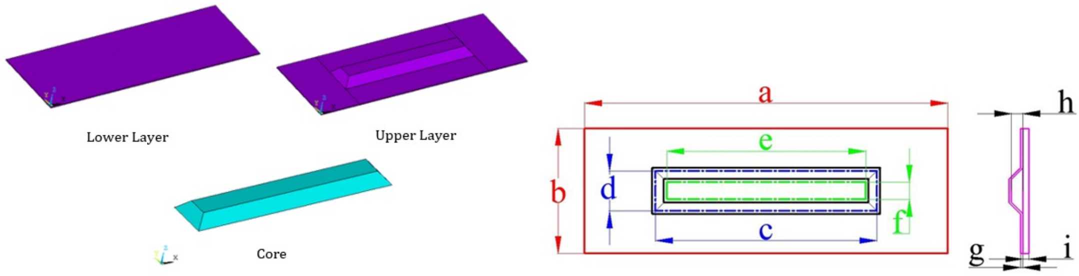
2.1 Panel properties and geometry

The studied panel here was proposed by Rifay et al. in [4], who developed a new procedure to produce composite plates of size $400 \times 140 \times 3$ mm^3 reinforced with a centrally located Omega feature. The sandwich hat-stiffened composite panel consists of three components: an upper composite layer, a lower composite layer, and a foam core separating them. (Fig. 2.a) [3] shows a illustration of the described layers.

The geometric parameters of this panel are presented in (Fig. 2.b) [3]. The core is made of Foam which has the following properties $(E = 1.5\,\mathrm{GPa}; \nu = 0.3\,)$. The composite parts are made of glass fiber-epoxy. The ply lay-up is $[90, 0, 90]$ with a total number of 3 plies, each having a thickness of 0.47mm.

2.2 Case study

According to experimental results in [4], this composite panel will act as an elastic one while the bending force is below $2000\,\mathrm{N}$. The hat stiffened panel is treated in omega case where a constant force $F = 1000\,\mathrm{N}$ is applied on the mid plane of upper layer, and two supports are



(a) (b)

Figure 2: a) Panel composition , b) Panel geometry.

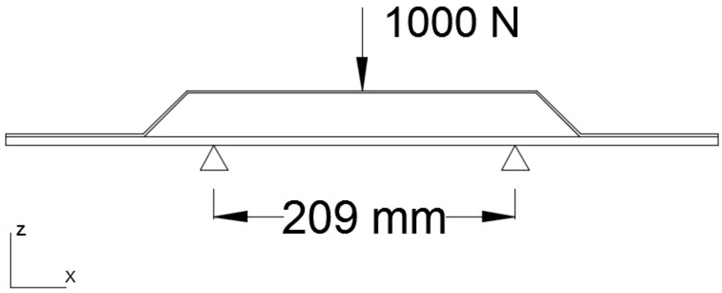


Figure 3: Omega loading.

placed in the opposite layer. Their inter distance is $209\,\mathrm{mm}$ as shown in (Fig. 3)[3]. (Table 1) shows the upper and lower bounds of variables vector. The mean vector of these bounds is very close to the dimensions of the experimental panel studed in [4]. The choice of these bounds in addition to the constant force $F = 1000\,\mathrm{N}$ ensures that we are still in the elastic region while the geometric parameters vary between variables bounds.

3 Optimization problem

This work aims at finding the best geometrical composition that maximizes the panel's rigidity with minimum weight. Let the geometrical variables vector be $x = \{c, d, e, f, g, h\}$, whose bounds are presented in (table 1). The parameters a and b are fixed to $406.4\,\mathrm{mm}$ and $104.34\,\mathrm{mm}$ respectively, while i is supposed to be equal to g as shown in (Fig. 2.b). The rigidity function R is presented in (1) where δ is the maximum panel deflection. The mass function M is obtained by (2), where V_c, V_f are the total volumes of composite parts and foam part respectively, andρ_c,

Table 1: Upper and lower bounds for variables vector

Variables	c	d	e	f	g	h
Lower bound x_l[mm]	234	44	200	18	1.24	11.4
Upper bound x_u[mm]	266	52	249	26	1.56	14.6

ρ_f are their corresponding densities.

$$R = \frac{F}{\delta} \tag{1}$$

$$M = V_c\rho_c + V_f\rho_f \tag{2}$$

3.1 Deterministic optimization

The deterministic problem is formulated in (3) as follows. Let F_{obj} be the objective functions vector, and x_l, x_u be the lower and upper variables vectors respectively. The minimum value of the rigidity is assumed to be equal to $600\,\mathrm{N\,mm^{-1}}$. This value in addition to the geometric constraint ensure the elastic behavior of the panel.

$$\begin{aligned} &\text{Minimize } F_{obj}(M(x), -R(x)),\\ &\text{Subject } to:\\ &\qquad R(x) \geq 600\\ &\qquad x_l < x < x_u \end{aligned} \tag{3}$$

3.2 Metamodeling

The calculation of the deflection using finite element simulation is exhaustive. In order to overcome this problem, a polynomial metamodel is constructed. The polynomial metamodel gives access to propagate uncertainty by analytic calculation. To construct this metamodel, 85 experiments are realized using ANSYS™. These 85 points are chosen using cubic face centered design of experiments. The obtained metamodel is shown in (4).

$$R = -130 + 82.3g + 0.3732dh + 0.3275dgh + 0.04731fgh^2 - 0.1517e - 0.05446d^2 \tag{4}$$

3.3 Reliability based optimization

Deterministic optimization produces results that are sensible to inputs variations and have a low reliability degree. In order to avoid this issue, the problem is studied under uncertainty with RBO formulation. The RBO aims to obtain optimums that respect the problem constraints with high reliability degree. Uniform distributions are associated to the problem variables and parameters. The parameters of these distributions are presented in (table 2), where x is the variables vector and $\rho = \{\rho_c, \rho_f\}$ is the densities vector . The objective functions in RBO are the mean of the mass $E[M]$, and the mean of the rigidity $E[R]$. The deterministic constraint is replaced by its quantile function $Q_k[R]$ with a reliability degree equal to k. The RBO is formulated in (5), where X and P are the vectors of random variables and random densities respectively.

Table 2: Uniform distribution parameters associated to each variable or parameter of the problem.

Variables and parameters	x	ρ
Lower bound [mm]	0.97x	0.95ρ
Upper bound [mm]	1.03x	1.05ρ

$$
\begin{aligned}
&Minimize\ F_{obj}(E[M(X,P)], E[-R(X)]) \\
&Subject\ to: \\
&\qquad Q_k[-R(X)] \leq -600 \\
&\qquad X = x + \chi_x \\
&\qquad P = \rho + \chi_\rho \\
&\qquad x_l < x < x_u \\
&\qquad x = \{d, e, f, g, h\}
\end{aligned} \tag{5}
$$

3.4 Optimization methods

To solve the RBO problem we adopte two different methods. The first is the combination of MC and NSGA2. Latin hypercube sampling (LHS) and common random number (CRN) [7] are used in MC with a sampling number $n = 10^6$ samples. For NSGA2, population the size is taken 70.

The second method is the combination of AC and NBI. The mean $E[g]$ and the standard deviation $\sigma[g]$ of a function $g(X)$ using AC are calculated by (6), where X is the variables vector, n is the number of variables, and $f_i(x_i)$ the probability density function of the random variable x_i.

In order to show the distribution of $[-R(X)]$, a large number of MC simulations containing 10^5 samples per simulation was executed. (Fig. 4) illustrates a sample of the obtained distributions. These distributions are found to be very close to normal distributions . This allows the estimation the quantile of rigidity using the equation of normal distribution defined in (7). The function $\phi^{-1}(k)$ is the inverse of cumulative distribution function of standard normal distribution corresponding to a probability k.

$$
\begin{aligned}
E[g(X)] &= \int_{-\infty}^{+\infty} .. \int_{-\infty}^{+\infty} g(X) \prod_{i=1}^{n} f_i(x_i) dx_i \\
Var[g(X)] &= E[g^2(X)] - E^2[g(X)] \\
E[g^2(X)] &= \int_{-\infty}^{+\infty} .. \int_{-\infty}^{+\infty} g^2(X) \prod_{i=1}^{n} f_i(x_i) dx_i \\
\sigma[g(X)] &= \sqrt{Var[g(X)]} \\
X &= \{x_1, x_2, .., x_n\}
\end{aligned} \tag{6}
$$

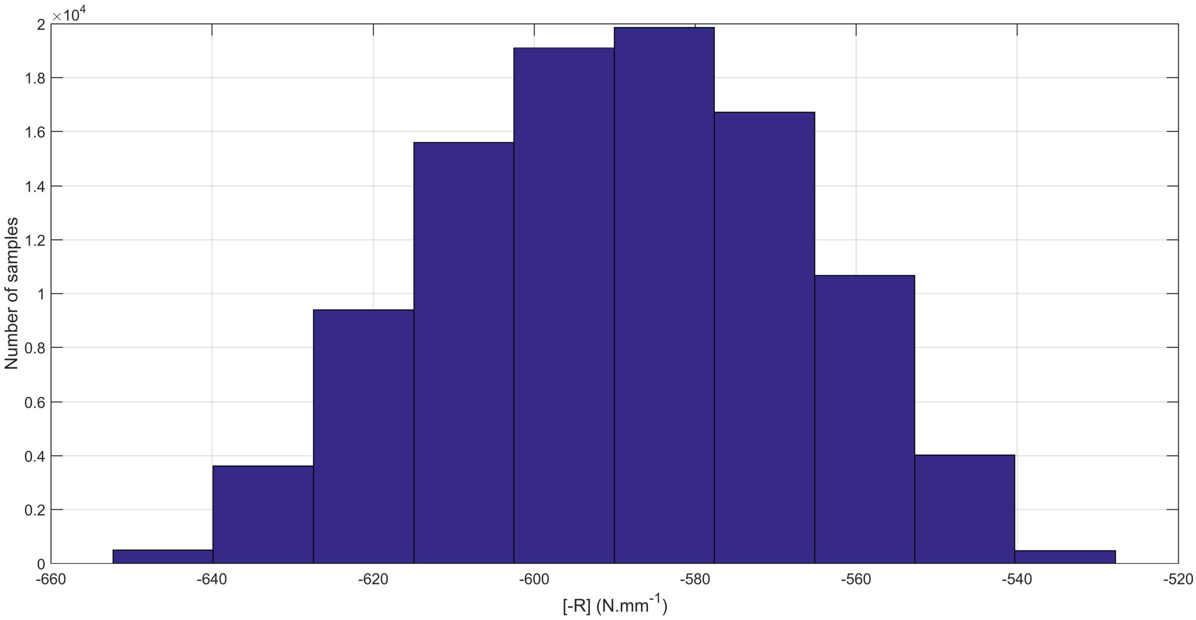


Figure 4: Distribution of $[-R]$

Table 3: Time and number of iterations consumed by NBI-AC, NSGA2-MC methods and deterministic optimization.

Method	Time	Number of iterations
NSGA2-MC	3400 s	7981
NBI-AC	18.6 s	21491
Deterministic	22.2 s	24646

$$\hat{Q}_k[-R(X)] = E[-R(X)] + \phi^{-1}(k)\sigma[-R(X)] \quad (7)$$

4 Optimization results

The problem is studied in deterministic and RBO cases. (Fig 5) shows the results of these cases. The RBO is evaluated using two reliability degrees 85% and 99.998%. The three obtained Pareto fronts are constructed using NBI where each one contains 25 points. The deterministic front covers a larger domain than the two RBO fronts. The RBO front that corresponds to 85% is larger than the one with reliability degree equal to 99.998%.

(Fig 6) compares the results of NBI-AC and NSGA2-MC indicating a dominance of NBI-AC. This dominance appears in the entire points of Pareto front. The results of NBI-AC are equally distributed along the entire front, whereas NSGA2-MC aren't. The time consumed and the number of iterations taken by each method are shown in (table 3). NBI-AC is faster than NSGA2-MC although its number of iterations is higher. It is worth noting that the execution time of NBI-AC is competitive to that of deterministic case.

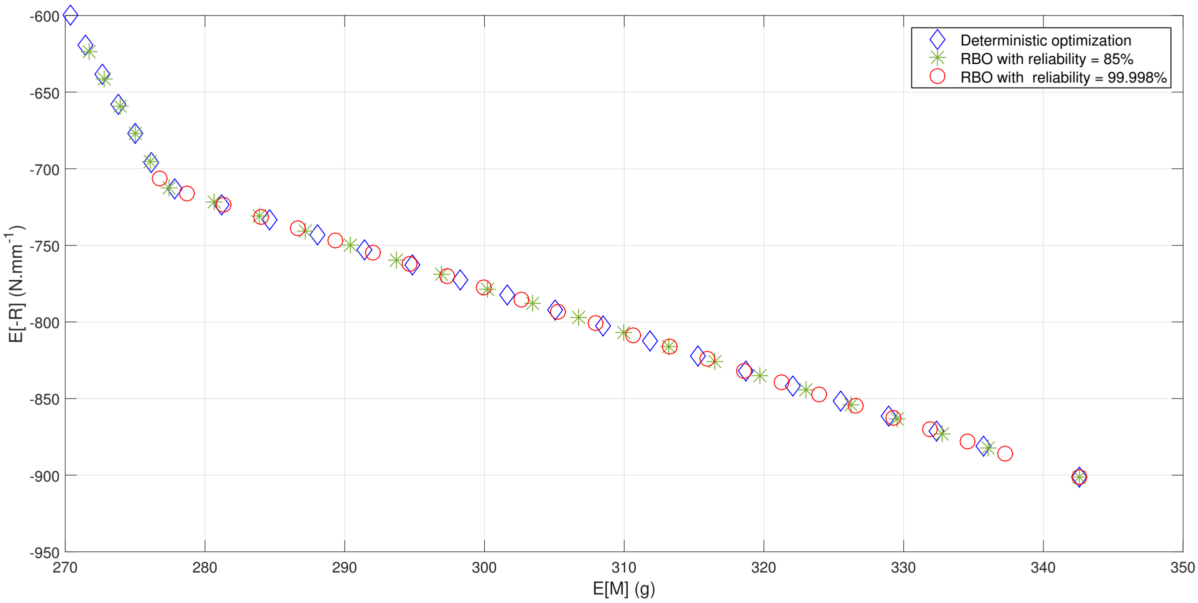


Figure 5: Comparison between deterministic and RBO Pareto fronts

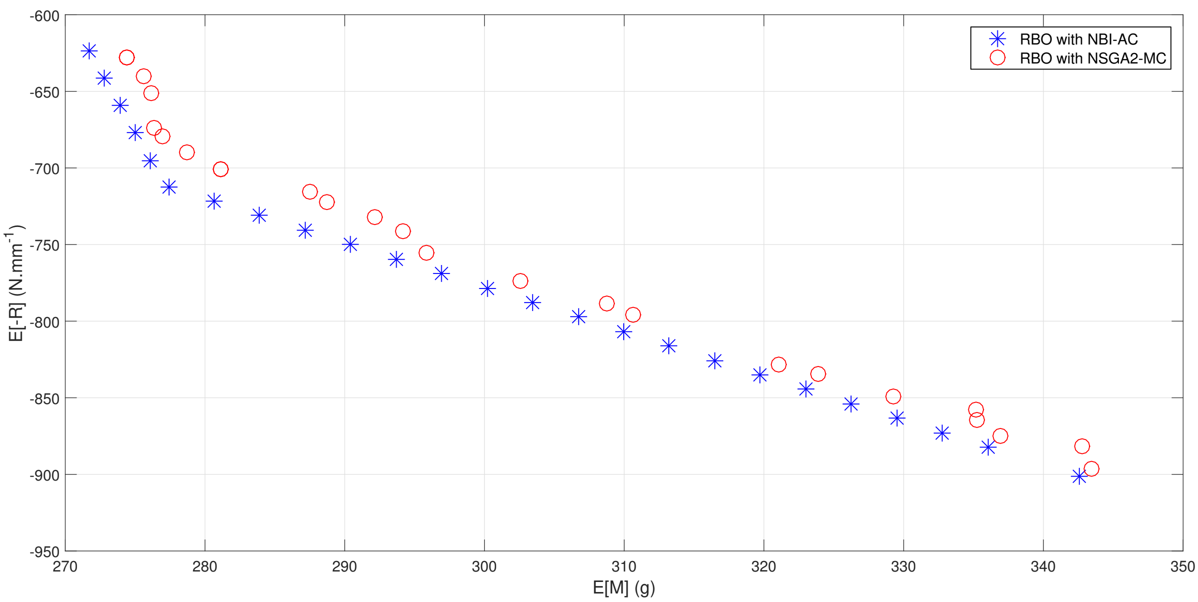


Figure 6: RBO Pareto fronts constructed by NBI and NSGA2 with reliability = 85%

5 Conclusions and Perspectives

In this work, an efficient NBI-AC method to solve RBO of a hat stiffened panel was described. This method is the combination of AC to propagate uncertainty and NBI to construct Pareto front. The results of this method are noted in the following. The hat stiffened panel problem should be treated in RBO, due to the influence of uncertainties in the deterministic case. Yet, the RBO results depend on the chosen degree of reliability. Moreover, the time required to compute the RBO is related to the applied method. It is found that NBI-AC requires a competitive time to that in the deterministic case. In addition, it produces an exact and equally distributed Pareto front. Whereas NSGA2-MC doesn't converge to the exact front and takes more time to reach it. The number of iterations in NBI is higher than that in NSGA2 due to the equality constraint used in the former method. This method can be applied to other problems especially when the Pareto front is convex. In the case of concavity of Pareto fronts, NBI can be replaced by Adaptive weighted sum (AWS) [8], that is more computationally complex. RO optimization is needed to evaluate the stability of the obtained results, however this will increase the complexity of the problem due to the addition of mass and rigidity standard deviations objective functions. In this case a multi-objective NBI function could be used [9].

References

[1] Y. Aoues, A. Chateauneuf (2010). Benchmark study of numerical methods for reliability based design optimization. Struct. Multidisc. Optim. 41(2), 277-?294.

[2] H. G. Beyer, B. Sendhoff,(2007). Robust optimization a comprehensive survey. Computer methods in applied mechanics and engineering, 196(33), 3190–321

[3] A. El Samrout, O. Braydi, R. Younes, F. Trochu, P. Lafon (2016).A new hybrid method to solve the multi-objective optimization problem for a composite hat-stiffened panel. THE 6TH INTERNATIONAL CONFERENCE ON METAHEURISTICS AND NATURE INSPIRED COMPUTING, META 2016, WILL HELD IN MARRAKECH (MOROCCO).

[4] M. Rifay (2010). Fabrication de panneaux auto-raidis par injection flexible (Doctoral dissertation, École Polytechnique de Montréal).

[5] K. Deb, A. Pratap, S. Agarwal, T. A. M. T. Meyarivan, (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE transactions on evolutionary computation, 6(2), 182-197.

[6] I. Das, J. Dennis (1996). Normal-Boundary Intersection: An Alternate Method for Generating Pareto Optimal Points in Multicriteria Optimization Problems (No. ICASE-96-62). INSTITUTE FOR COMPUTER APPLICATIONS IN SCIENCE AND ENGINEERING HAMPTON VA.

[7] W.N. Yang, B.L. Nelson, (1991). Using common random numbers and control variates in multiple-comparison procedures. Operations Research, 39(4), 583-591.

[8] I.Y. Kim, O.L. De Weck, (2006). Adaptive weighted sum method for multiobjective optimization: a new method for Pareto front generation. Structural and multidisciplinary optimization, 31(2), 105-116.

[9] R.D.S. Motta, S. M. Afonso, P.R. Lyra, (2012). A modified NBI and NC method for the solution of N-multiobjective optimization problems. Structural and Multidisciplinary Optimization, 46(2), 239-259. Chicago