Localized Boundary Layer Receptivity
To a Free Stream Acoustic Disturbance

Y. BANDADI, A. SBAIBI**

Faculté des Sciences et Techniques Mohammedia
Université Hassan II Casablanca
** Corresponding Author: ahmed.sbaibi@univh2m.ma

Résumé :

La réceptivité acoustique de la couche limite de Blasius en présence d'une rugosité de surface bidimensionnelle est étudiée numériquement. Il est montré que, l'excitation externe sous forme d'onde acoustique amplifie le mode propre instable Tollmien-Schlichting (T-S) de la couche limite. Cette amplification dépend de nombreux paramètres tels que la forme géométrique de la bosse, sa hauteur et aussi l'amplitude acoustique. L'emplacement de l'inhomogénéité de la surface est déterminé à partir de la courbe marginale d'instabilité au niveau la branche inférieure, dérivée des équations d'Orr-Sommerfeld.

Abstract:

Acoustic receptivity of a Blasius boundary layer in the presence of two dimensional surface inhomogeneity is investigated numerically. It is shown that, an efficient conversion of the acoustics input to an unstable eigenmode of the boundary layer depends on many parameters: the shape of the hump, the height and also on the acoustic amplitude. The location of the surface roughness is dictated by the requirement of the instability wave number at the lower branch of the stability curve derived from the Orr-Sommerfeld equations.

Mots clefs: Acoustic, receptivity, Instabilities, Tollmien-Schlichting, Boundary Layer flow

1   Introduction

Understanding the transition from laminar to turbulent flow in the boundary layer is of great importance for scientists as well as for engineers. For such transitional boundary layers, the skin friction and the heat transfer coefficient change dramatically as the boundary layer undergoes transition from laminar to turbulent flow. The relevant mechanism to this transition is not easy to reveal because various physical mechanisms may initiate the transition process. However, the amplification of the instabilities are significantly influenced by external disturbances in the form of

perturbation such as vortical, entropy or acoustic fluctuation around the base flow. The classical process of laminar-turbulent transition is subdivided into three stages: receptivity, linear eigenmode growth and non-linear breakdown to turbulence. The receptivity process in the boundary layer starts from the moment an external disturbance enters the boundary layer and generates instabilities. In the case where the disturbance is an acoustic wave, energy is transferred from the acoustic wave to Tollmien-Schlichting (TS) instabilities through wavelength scattering. Goldstein [1] identifies two kinds of TS wave generation: generation due to the leading edge of the boundary layer growth and generation by a change in the surface geometry. The generic problem of this second kind of TS wave generation is that of a relatively thin body with a small local region of large surface curvature referred to as "Interaction region" and taken to be downstream form the leading edge at Lc, where Lc is the distance to neutral stability curve. We focus in this paper on the receptivity in this interaction region of a subsonic boundary layer to a free stream acoustic perturbation.

The heights of the roughnesses involved in the receptivity study are usually assumed to have a size much smaller than the boundary layer thickness in order to cause only a small distortion. However, as the height increases stability characteristics may be modified due to the change that may occur in the mean flow.

The first to consider the presence of the boundary layer in the presence of an isolated 2D roughness was Klebanoff and Tidstrom [2] in their experimental work for h/δ^* around 0.7 (δ^* being the boundary layer displacement thickness). Theoretical investigation includes but not limited to the work of Nayfeh and Ashor [3], Crouch [4]. Most theoretical investigations are based on the assumption of parallel flow. A Parabolised Stability Equations PSE approach was used by Wie and Malik [5] in order to account for the inherent non-parallel flow in the vicinity of the roughness. Tullio et al [6] assessed the triple-deck theory by comparing its results to numerical simulations of the Navier Stokes when investigating the receptivity of subsonic compressible boundary layer in the presence of a small surface inhomogeneity.

2 Methodology

2.1 Physical parameters of the simulation

The numerical experiment is largely inspired by the experimental work of Saric et al. [7] performed in wind tunnel during their study of acoustic receptivity due to a 2-D roughness on a flat plate with a minimum leading edge effect. The roughness is located at 0.46m of the leading edge which has a Reynolds number $U.\delta^*/\nu=1015$ where δ^* is the BL displacement thickness, the frequency parameter of the problem given by the linear theory is $F=49.33\ 10^{-6}$. Figure1 gives a rough sketch of the experiment and the location of the surface roughness.
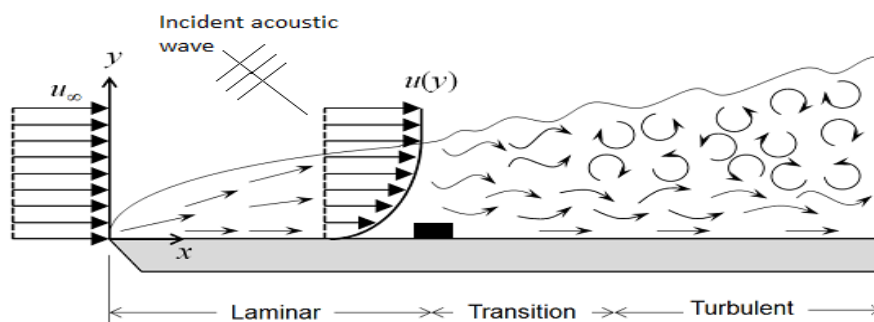

Figure.1: Schematization of the problem

2.2 Conservations Equations

The conservation equations are similar in their flux form to the pseudo-compressible form of the Navier-Stokes equations introduced by Chorin [8]

$$\frac{\partial q}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = \frac{1}{Re}\left[\frac{\partial(f_\nu)}{\partial x} + \frac{\partial(g_\nu)}{\partial y}\right]$$

The variables vector q and the inviscid flux vectors are

$$q = \begin{bmatrix} p_\rho \\ u \\ v \end{bmatrix} f = \begin{bmatrix} \beta u \\ p_\rho + u^2 \\ uv \end{bmatrix} g = \begin{bmatrix} \beta v \\ uv \\ p_\rho + v^2 \end{bmatrix}$$

$$p_\rho = \frac{p - p_0}{\rho_0}$$

The viscous flux vectors are:

$$f_\nu = \begin{bmatrix} 0 \\ \frac{\partial u}{\partial x} \\ \frac{\partial v}{\partial x} \end{bmatrix} \qquad g_\nu = \begin{bmatrix} 0 \\ \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial y} \end{bmatrix}$$

In the above equations Re is based on the plate length and β is the "finite compressible factor" . The correct physical value of β is 1/Ma (Ma being the Mach number) as opposed to computationally convenient values typical of pseudo-compressibility solutions.

2.3 Numerical Solution

The conservation equations are solved using a time accurate Navier Stokes code (Manno et al. [9]). Viscous terms are approximated using central differencing while the convective terms are discretized using a third order MUSCL TVD scheme Van leer in conjunction with Riemann solver (Sbaibi et al.[10]). Two options are used for the time advancement. An implicit, approximate factorization, time advancement is used only to accelerate to steady state conditions that characterize the base flow over which a transient component is added and an explicit, three-step, second order Runge-Kutta algorithm is used to accurately simulate the transient computations.

The computer code was extensively validated for a wide range of steady and transient flow situations including some fluid-acoustics problems Manno et al.[9], Reitsma et al. [11].

The computer domain of (1275x52) cells includes the leading edge of the plate and an exit zone used as buffer zone. A non-uniform griding is performed in order to capture the fine structure near the surface roughness. The inflow boundary condition is a first order non-reflective boundary based on perturbations around the Blasius velocity profile which includes the acoustic source as shown in the following equations:

$$p_\rho = 1 + a.cos(kx - \omega t) \quad ; \quad u = 1 + (k.a).cos(kx - \omega t)/\omega \quad ; \quad v = 0$$

Where **a**, **ω** and **k** are respectively the acoustic amplitude, the acoustic frequency (**ω**=37.35 s^{-1}) and the number wave.

The boundary condition on the top sets **u** and **v** velocity gradients to zero and with the pressure calculated from the appropriate characteristic compatibility relations. A non-reflective outflow boundary condition was imposed on exit zone.

3   Results

3.1   Identification of T-S waves

The first numerical investigation of the T-S waves deals with a surface roughness of rectangular shape that simulates the Mylar strip used by Saric et al. [7]. The width is set to λ_{TS}/2. A converged steady state without any acoustic source is obtained with a large CFL over 20000 time steps using the implicit approximately factored algorithm and then the calculation is restarted using an explicit Runge-Kutta method with a low CFL number. This converged steady state will provide a baseline against which to compare the unsteady solution. During the second stage of the explicit computation, the acoustic source is triggered. The acoustic wave is allowed to cross the computation domain with few wavelengths. The obtained solution is a blend of the base flow; the unsteady stokes component as well as the TS component. The Stokes solution is obtained when using the acoustic source that interacts with the boundary layer flow of a homogeneous flat plate (without roughness) as described by (Worner et al. [12]. A sample of the u'$_{TS}$ at a fixed value of y/Lref =0.00078 for different x location is shown in figure 2. The exponential amplification of the u'$_{TS}$ component is clear from plot. Likewise, a plot of the u'$_{TS}$ versus the y elevation is presented in Figure 3. These two plots exhibit the same behavior and compare favorably to the largely published results. The most unstable λ_{TS} obtained from Figure.2 is equal to 0.054 which is similar to the analytical value based on the linear stability theory. Plots similar

to the one presented in figure 2 allows the computation of the spatial amplification factor α_{TS} by frequency inverting method or by simply taking the slope of the curve (Log (u'$_{TS}$)$_{max}$ versus the x position).
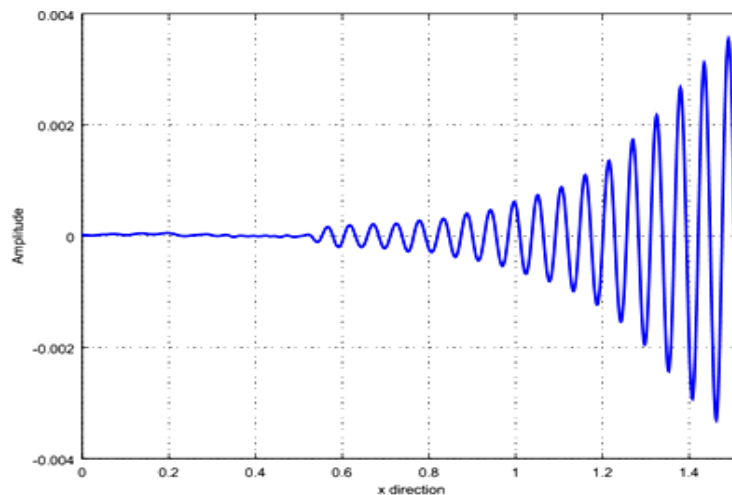

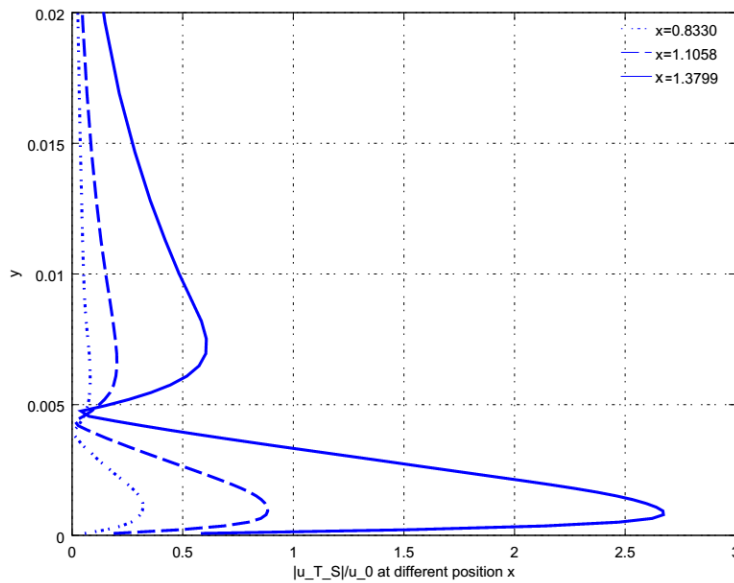Figure 2: T-S perturbation profile of u velocity

Figure 3: Amplitude of T-S waves at different x positions (rectangular roughness)

The average spatial amplification factor of the TS wave is given by:

$$\alpha_{TS}=1/(x2 -x1) \, \text{Ln} \, (A_2 / A_1)$$

where A_1 and A_2 is the wave amplitude at x_1 and x_2 respectively.

3.1 Influence of the roughness geometry

In order to highlight the impact of the surface roughness geometry, three different shape functions were considered (rectangular, cosine and exponential). Each one was extensively studied with different heights. It should be mentioned that all the surfaces considered have the same width which is set to $\lambda_{TS}/2$. The effect of the roughness width was investigated in a companion paper by Bandadi and Sbaibi [13] where it was found that a width of $\lambda_{TS}/2$ gives the most receptive results for the three surface shapes considered. plots of the three surface shapes are presented in Figure 4.
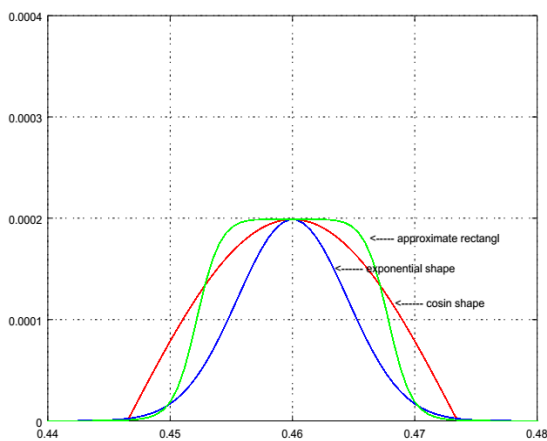

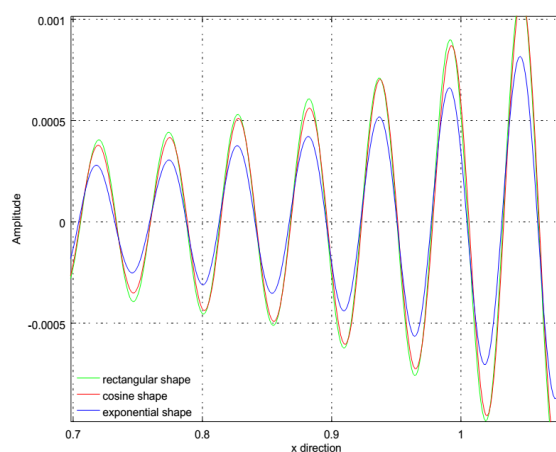Figure.4: visualization of roughness shapes

Figure.5: T-S perturbation profile of u velocity triggered by the different shapes
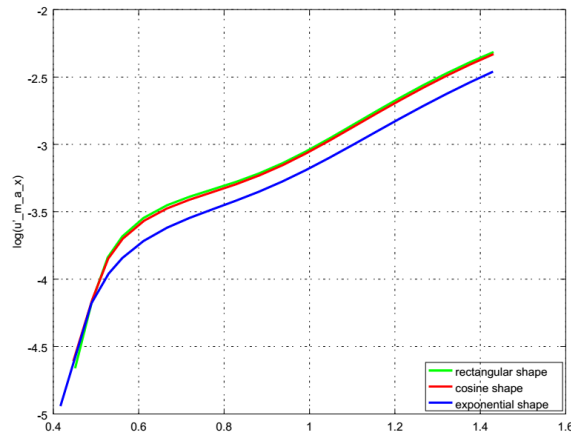
Figure.6: Amplitude slope of a TS waves for the different shapes

The impact of the shape function on the behavior of the TS wave is given on figure 5. Only the amplitude of the TS is influenced. The rectangular and the cosine give close results while the amplitude of the TS obtained by the roughness with an exponential shape is the lowest. The wavelength of the TS wave is the same for the three shapes. This was expected since the TS wave development depends only on the flow characteristics at the given location.

The logarithm of amplitude of the TS wave downstream the surface inhomogeneity is given on figure 6 for different shapes. Downstream the inhomogeneity location, the curves are nearly parallel. The identical slopes are a strong indication that the amplification factor is a shape free dependent.

The impact of the roughness height on the amplitude of the TS wave at x=0.9 is reported on figure 7, 8, and 9 respectively for the cosine, the rectangular and the exponential surface geometry where the amplitude is plotted against the scaled height h/δ*. Note that the Amplitude varies linearly up to some specific value of h/δ* and then starts to deviate from the linear curve. This deviation is more pronounced for the rectangular and the cosine shape.

The linear behavior for the exponential shape extends over a large interval of height. In fact, Saric et al. [7] mentioned that nonlinear effects become more dominant when the reattachment length of the separated bubble exceeds the hump height.
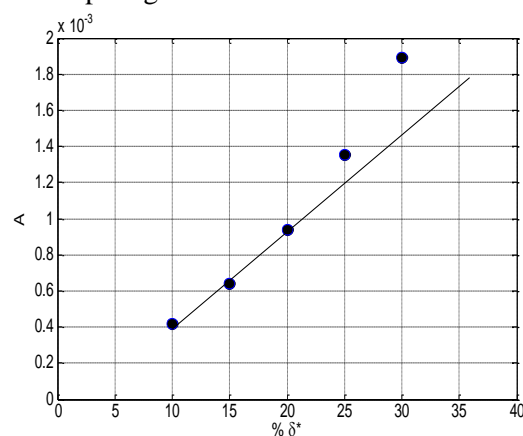

Figure.7: influence of height on the amplitude of the TS waves triggered by a rectangular bump
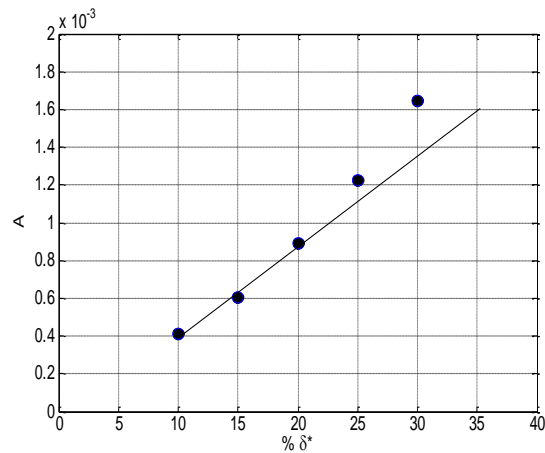
Figure.8: influence of height on the amplitude of the TS waves triggered by a cosine bump
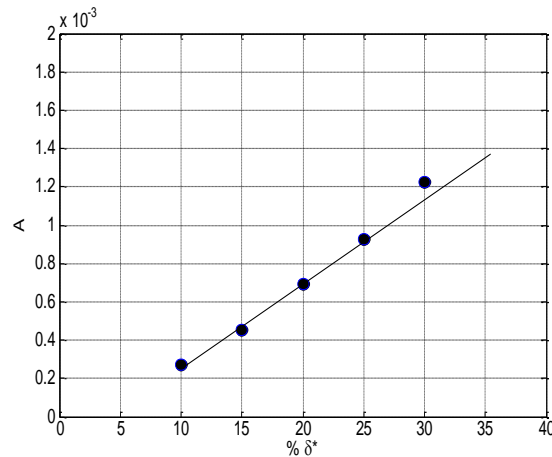

Figure.9: influence of height on the amplitude of the TS waves triggered by an exponential bump

3.2   influence of the acoustic forcing amplitude

The assessment of the impact of the acoustic forcing amplitude on the amplitude of the TS wave was considered for the rectangular shape. Different sound levels were used to study the receptivity of the boundary layer. It was found that the amplitude varies linearly with the sound amplitude as shown on figure 10. This result is in good agreement with the experimental work carried out by Leehey and Shapiro [14] who produced a nearly plane acoustic waves by placing a large acoustic speaker upstream the leading edge of a flat plate. They found that the TS amplitude increased linearly with the amplitude of the imposed disturbance.
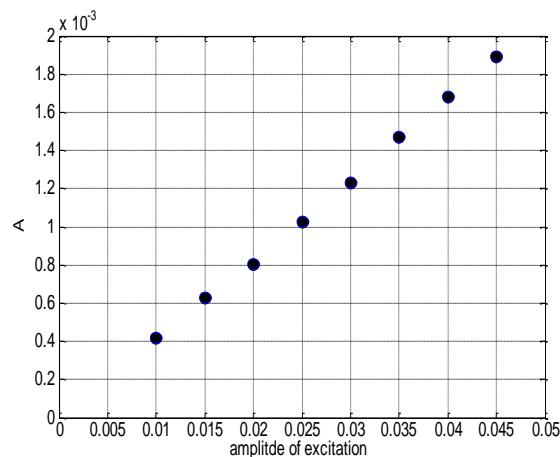

Figure.10: influence of amplitude of acoustic excitation on amplitude of TS waves

4   Conclusion

Numerical investigation of a subsonic boundary layer receptivity to a free stream acoustic disturbance in the presence of a 2D surface inhomogeneity was conducted. The effect of the acoustic forcing as well as of the hump height on the Tollmien Schlichting amplitude was investigated. It was found the TS amplitude responds linearly the variation of the acoustic perturbation amplitude. However, this TS amplitude varies linearly with the bump height only for lower value of the height compared to the boundary layer displacement thickness. For higher considerable values of the height, the amplitude of TS shows a nonlinear behavior. The nonlinear threshold value of the height depends on the shape function of the hump. The results presented in this paper are in good agreement with the published results when it applies. Also, this numerical study had put forward the superiority of the accurate computational solver based on the advanced numerical methods inspired from the compressible flow solvers. The finite compressible method used here seems to be promising in simulating some other aero_acoustic problems without any assumption as done in other theoretical methods or in incompressible flow solvers.

References

[1] Goldstein, M. E. Scattering of acoustic waves into Tollmien-Schlichting waves by small streamwise variations in surface geometry. J. Fluid Mech. 154, 1985, pp 509–529.

[2] Klebanoff, P. S., Tidstrom, K. D., and Sargent, L. M., "The Three-Dimensional Nature of Boundary-Layer Instability," Journal of Fluid Mechanics, Vol. 12, 1962, pp. 1-34.

[3] A. H. Nayfeh and O. N. Ashour, "Acoustic receptivity of a boundary layer to Tollmien–Schlichting waves resulting from a finite height hump at finite Reynolds numbers",Physics of Fluids 6, 3705 (1994)

[4] J. D. Crouch, "Localized receptivity of boundary layers," Phys. Fluids A 4, 1408 (1992).

[5] Wie, Y. & Malik, M. R. 1998 Effect of surface waviness on boundary-layer transition in two-dimensional flow. Comput. Fluids 27, 1998, 157–181.

[6] Tullio, N., and Ruban A.,I.,A numerical evaluation of the asymptotic theory of receptivity for subsonic compressible boundary layers, JFM 196, 2015

[7] W. Saric, J. Hoos, and R. Radeztsky, "Boundary-layer receptivity of sound with roughness," Boundary Layer Stability and Transition to Turbulence, edited by D. C. Reda, H. L. Reed, and R. Kobayashi (American Society of Mechanical Engineers, New York, 1991), FED No. 114, pp. 17-22.

[8] A.J. Chorin, A numerical method for solving incompressible Visqueuse flow problems, Journal of computational physics. 1967.

[9] V.P.Manno, S.H.Reitsma, and T.F.Tureaud, Developing Numerical Techniques for Solving Low Mach Number Fluid Acoustic Problems. AIM Journal, 1993, pp.1984-1991

[10] A.Sbaibi and V.P.Manno. On the Accuracy of Upwind and Symmetric TVD Schemes in Simulating Low Mach Number Flow. IJCFD. 2000. Vol. 13. pp. l25-142

[11] S.H.Reitsma, V.P Manno, and T.F Tureaud, Numerical Simulation of Receptivity Phenomena in Transitional Boundary-Layer Flows, AIAA Journal, 1997,Vol. 35, No. 5

[12] A S.Worner. Herr, S. Wagner, Y.S. Kachanov, Study of the acoustic receptivity of a blasius boundary layer in the presence of a surface non-uniformity. European Congress on Computational Methods in Applied Sciences and Engineering. 2000. Sep. 11-14, barcelone, Spain

[13] Bandadi Y. and Sbaibi, A. Subsonic Boundary Layer Receptivity to Free Stream Acoustic Perturbations 13 ème Congrès de Mécanique 11 - 14 Avril 2017 (Meknès, MAROC)

[14] Leehey P. and Shapiro P.J, Leading edge effect in laminar boundary layer excited by sound. Proc. IUTAM Symp. On Laminar-turbulent transition, Springer-Verlag 232-242, 1980