Multi-Objective and Non-deterministic Optimization of Vehicle Restraint Systems

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Abstract – Procedures for Multi-Objective Non-deterministic Optimization (MONO) of complex engineering systems are discussed and the optimization of a Vehicle Restraint System (VRS) is realized with Finite Element (FE) simulation. In the previous studies: performances of a VRS have been tested by crashing with a vehicle and the FE model of the crash test has been created; the critical factors whose uncertainties contribute the most to the performance uncertainties of the VRS have been identified through sensitivity analysis. In this article: the inputs' space of design variables and outputs uncertainties are studied to define design intervals and the constraints of optimization; Kriging interpolation method is used to create the surrogate model and the precision of the surrogate model is estimated with a new approach; uncertainties of the critical factors are considered and the VRS is optimized through MONO with the surrogate model; performances of the optimized device are evaluated under different crash conditions.

Key words: crash simulation / multi-objective optimization / robust design / uncertainty analysis / sensitivity analysis / Vehicle Restraint Systems

1 Introduction

Vehicle Restraint Systems (VRS) are specially designed to restrain an errant vehicle by dissipating or absorbing the impact energy and redirecting the vehicles to reduce crash accident severity and protect the roadside equipment [1]. Before being installed on the roadside, a VRS must be tested by crashing with vehicles to evaluate its performances for severity reduction in traffic accident. The installation conditions and crash conditions of the VRS are innumerable. In Europe, the norm EN1317 [2, 3] define the containment levels, the relative standardized test conditions and performance criteria of the VRS. Uncertain factors in the crash test of the VRS complicate the validation of the device. It is economically impossible to evaluate the robustness of a design through the crash tests. Dynamic simulation is used in the design of the VRS. By defining the model input parameters at different possible levels and taking samples with Design of Experiment (DOE), C.Goubel [4] analyzed qualitatively the robustness of a VRS with dynamic simulations. As for the optimization of a VRS, the situations are much more complicate:

- Many engineering optimization problems have multiple nonlinear objectives and constraints, mixed continuous-discrete design variables;
- Uncertainty of parameters is inevitable and can significantly degrade the performance of a design;
- The norm EN1317 specified the test conditions of the device. But in reality, the accidents occur at different crash situations.

The designs with multiple objectives considering model uncertainties are "Multi-Objective and Non-deterministic Optimization (MONO)". Multi-objective optimization, i.e. Pareto optimization, is a multiple criteria decision making process. Pareto efficiency is a state of allocation of resources in which it is impossible to make any one individual better off without making at least one individual worse off. Non-deterministic optimization [5] aims to optimized the performances of a system, and maximize the robustness of the design in the same time.

The challenges for MONO of complex engineering systems include: high calculation cost of model simulation; numerous uncertain factors in the models; lack of information about the design space and model uncertainties;

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diversity of the crash conditions. The procedures are discussed for MONO of engineering systems; MONO of a VRS is realized and the optimized design is evaluated under various accident situations.

2 Procedures for MONO

2.1 System modeling & Simplification

Thousands of model runs are needed in the computer aided optimization problems. Numerical simulations are usually of high calculation cost. A model of high accuracy and relatively low calculation cost is needed, and the system modeling & simplification are of great importance.

2.2 Model Sensitivity Analysis

Although many uncertain factors may exist in an engineering model, only a few of them are influential on model performance. Optimizations considering all the uncertain factors may increase greatly the number of simulations. Sensitivity Analysis (SA) [6] is a natural previous & next step of robust optimization, especially for the applications where it is critical to identify the factors whose uncertainties have great influence on system's performances. The uncertain factors can be reduced by fixing the non-influential factors and considering only the uncertainties of critical ones in MONO.

2.3 Define of optimization Objects and Constraints

For a model with inputs vector **x**, uncertain factors vector **p**, outputs vector **F(x,p)** of m dimensions, inequality constraints g_i, equality constraints h_j, design variables intervals $[\mathbf{x}^{under}\ \mathbf{x}^{upper}]$, the multi-objective optimization problem is formulated as:

$$\min_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{p}) = \left[F_1(\mathbf{x},\mathbf{p}),...,F_m(\mathbf{x},\mathbf{p})\right]$$
$$s.t: \begin{cases} g_i(\mathbf{x},\mathbf{p}) \le 0, i=1,2,...,k \\ h_j(\mathbf{x},\mathbf{p}) = 0, j=1,2,...,l \end{cases} \quad (1)$$
$$with: \mathbf{x}^{under} \le \mathbf{x} \le \mathbf{x}^{upper}$$

Uncertain factors are fixed to their mean values $\mathbf{p_0}$ in deterministic optimization. Influenced by uncertainties of **p**, the objects and constraints need to be redefined in MONO. Considering model feasibility [7], the constraints are defined:

$$s.t: \begin{cases} \max\limits_{\mathbf{p}} g_i(\mathbf{x},\mathbf{p}) \le 0 \\ \max\limits_{\mathbf{p}} \left|h_j(\mathbf{x},\mathbf{p})\right| \le \xi_j \end{cases} \quad (2)$$

or

$$\begin{cases} \Pr\left(g_i(\mathbf{x},\mathbf{p}) \le 0\right) \ge C_i, i=1,2,...,k \\ \Pr\left(\left|h_j(\mathbf{x},\mathbf{p})\right| \le \xi_j\right) \ge D_j, j=1,2,...,l \end{cases} \quad (3)$$

Eq. (2) ensures the feasibility of inequality constraints g_j and restrains the deviation of equality constraints h_j under the limits ξ_j. Eq. (3) defines the model feasibility with probability and statistics approach, and restrains that the feasible probabilities of the constraints exceed C_i and D_j.

The outputs can be redefined as follows for the purpose of objective robustness in non-deterministic design:

$$\min_{\mathbf{x}} \left(\max_{\mathbf{p}} \mathbf{F}(\mathbf{x},\mathbf{p}) \right) \quad (4)$$

or

$$\min_{\mathbf{x}} \left(\mathbf{F}(\mathbf{x},\mathbf{p}_0) + c\,\boldsymbol{\sigma}_{\mathbf{p}}(\mathbf{F}) \right) \quad (5)$$

or

$$\min_{\mathbf{x}} \mathbf{F}(\mathbf{x},\mathbf{p}_0)\ s.t \max \mathbf{F}(\mathbf{x},\mathbf{p}) - \mathbf{F}(\mathbf{x},\mathbf{p}_0) \le \boldsymbol{\Delta}_{\mathbf{F}} \quad (6)$$

Eq. (4) minimizes the possible maximum output values of the designs. Eq. (5), with statistics method, calculates the outputs distributions for each design, and minimizes the sum of normal value and deviation value with scale factor c. Eq. (6), with robust optimization, selects the optimized solutions with outputs deviations under the defined limit $\boldsymbol{\Delta}_{\mathbf{F}}$ [8].

2.4 Define of design space

Before the optimization process, the intervals of design variables $[\mathbf{x}^{under}\ \mathbf{x}^{upper}]$ need to be selected. Initially, no information about inputs space is known and the intervals are defined artificially. The predefined design space may not cover the whole designs which are Pareto efficient or covers the regions away from the optimal solutions which will cause unnecessary simulations for optimization. DOE takes samples and runs the simulations cross the whole inputs space. The rationality of the predefined inputs space can be checked through DOE, and then the design space could be redefined to cover the all possible Pareto efficient inputs combinations and to take samples around the optimal points.

2.5 Metamodelling

Engineering simulations are usually of high calculation cost and metamodelling is used to create the surrogate model. DOE and model runs cross the whole inputs space are needed to clarify the relationship between outputs and inputs. For a model of n_x design variables and n_p uncertain factors, the model inputs dimension is n_x+n_p and large number of samples is required in order to ensure accuracy of the surrogate model. Latin Hypercube Sampling (LHS) is a widely used DOE technique for model performance study [9,10]. Stochastic interpolation with Kriging method gives unbiased prediction of the intermediate values and is used in the domain of simulation experiment [11,12].

2.6 Optimization

Optimization algorithms [13,14] (such as Genetic Algorithm, Particle Swarm Optimization, Simulated

Annealing, etc.) can be used for multi-objective design and have been integrated in mathematical software.

2.7 Verification of surrogate model

Surrogate model needs to be validated before being used to replace the simulation model. However, it's hard to define the accepted error of surrogate model in MONO problems:

- Both the uncertain factors and the design variables are inputs of the model in MONO problems, and the surrogate model needs to be validated across the whole inputs space of high dimension;
- Precision of a surrogate model may influence the Pareto efficiency of designs [8]: in **Fig. 1**-left, both *i* and *j* are Pareto efficient predicted with the surrogate model, but in fact design *i* is more preferable for both outputs criteria; in **Fig. 1**-right, design *i* is preferable than *j* predicted by the surrogate model, in fact it is exactly the opposite.

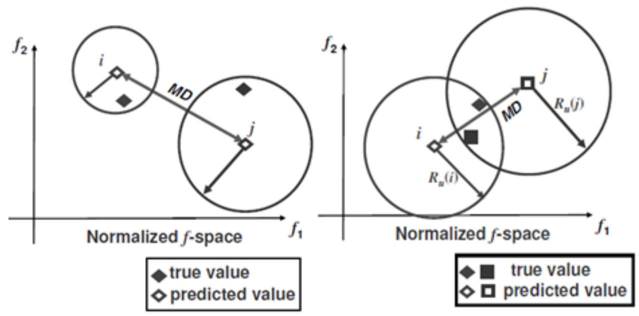

Fig. 1 Failure of design selection with surrogate model, true value: outputs calculated by model simulation; predicted value: outputs predicted with surrogate model; R_u: error region of surrogate model

Li [8] created the criteria to examine if Pareto efficiency of a design could be influenced due to the error of the surrogate model---The efficiency influenced designs will be calculated with simulations and non-influenced designs will be predicted with surrogate model during optimization process. Li's method is efficient, but efforts are needed to integrate this approach into optimization algorithm.

Here a practical way is proposed to ensure the accuracy of surrogate model in MONO problems and the validation of the surrogate model is shown in **Fig. 2**:

- The surrogate model will firstly be created with reasonable number of samples and be used for system optimization;
- The Pareto efficient designs $\mathbf{X}_{0i}=(x_{0i}^1, x_{0i}^2,\ldots,x_{0i}^k,)$ are then selected, where k is the number of design variables, i=1,2,…,n, for the optimization design with n Pareto efficient solutions;
- The input intervals of the optimal designs are studied and defined: $[\min(x_0^j)\ \max(x_0^j)]$, where $\min(x_0^j)=\min(x_{01}^j, x_{02}^j,\ldots, x_{0n}^j)$, $\max(x_0^j)=\max(x_{01}^j, x_{02}^j,\ldots, x_{0n}^j)$, with j=1,2,…,k;
- Additional samples will be taken and simulated in the new defined input intervals $[\min(x_0^j)\ \max(x_0^j)]$, The surrogate model is then updated with the new samples and the model will be optimized with the new surrogate model; new optimal designs $\mathbf{X}_{1i}=(x_{1i}^1, x_{1i}^2,\ldots,x_{1i}^k,)$ and updated intervals $[\min(x_1^j)\ \max(x_1^j)]$ are obtained;
- The samples refinement for accuracy improvement of surrogate model and the system re-optimization are repeated, and final optimal designs are obtained when they are no longer influenced by the refinement of samples.

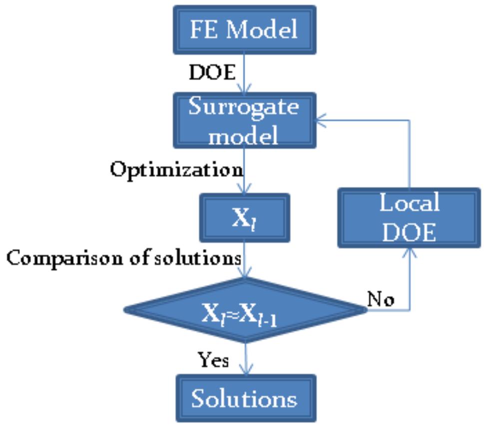

Fig. 2 Validation of the surrogate model

Instead of taking additional samples cross the whole inputs space, refinement of samples around the potential optimal solutions which will greatly reduce additional samples required to create an accurate surrogate model.

More efforts are needed to normalize the metamodeling & optimization process: the number of samples initially taken should be proportional to the number of design variables, and their relationship could be created; the conditions when the optimal solutions are no longer influenced by the refinement of samples need to be standardized.

3 MONO of VRS

3.1 VRS & Vehicle crash model

A steel VRS of containment level N2 were tested under TB32 test conditions [15]. The tested VRS is composed with the beam Rail, Spacer, support Post and the components are assembled by bolt connections. The tested VRS is illustrated in **Fig. 3**. A guided vehicle of 1431 kg in mass struck the VRS at speed 113.6km/h, at an angle of 20 °.

The FE model is illustrated in **Fig. 4**. Considering the magnitude of components' deformations, the crash model was modeled and simplified in the four aspects with FE program LS-DYNA [16]:

1) Coarse mesh with refinement for the parts with large deformations;
2) Simplification of VRS continuations at both ends of

the barrier with spring elements to apply the boundary constraints;

3) Detailed modeling of the soil for the parts with large deformation and its replacement by spring elements for the others soil parts;
4) Bolted joints simplification with spring elements for rigid connections.

Fig. 5 compared the experimental test and simulation results at different impact time. The FE model has been proved to be of acceptable accuracy. Different from detailed modeling which may take days of time for the crash simulation, the simplified model takes only about 5 hours for a single model run.

Fig. 3 The VRS evaluated by crash test with vehicle

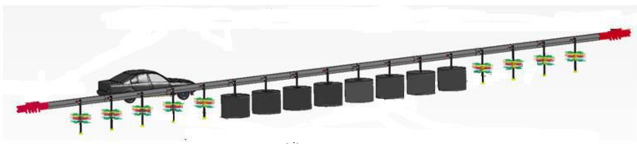

Fig. 4 Numerical model for crashing test of the VRS

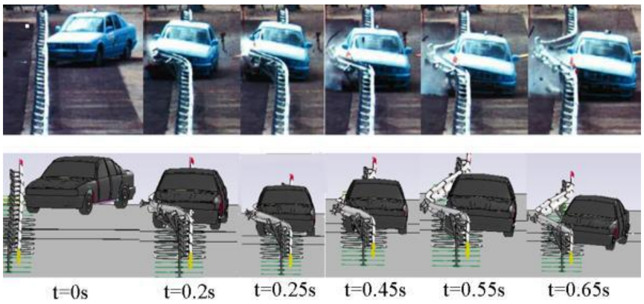

Fig. 5 Crashing test and simulation at different impact time

3.2 Sensitivity Analysis of VRS

SA of the VRS was realized in the previous research [17], 11 uncertain variables were chosen:

- Uncertain factors of material mechanical properties: Rail Yield strength (*RY*), Rail young Modulus (*RM*), Spacer Yield strength (*SY*), Spacer young Modulus (*SM*), Post Yield strength (*PY*) and Post young Modulus (*PM*);
- Tolerances of fabrication: Rail Thickness (*RT*), Spacer Thickness (*ST*), Post Thickness (*PT*);
- Uncertainties in installations: Soil bulk Modulus (*SoilM*), Bolt connection Pre-load (*BP*).

Characterization of uncertainty in inputs is an important part of SA as it determines both uncertainty in model outputs and sensitivity of outputs to the elements of uncertain input factors. To simplify the characterization of uncertainty, the uncertain parameters are defined with the classic 'crude' method by supposing they have normal distributions, and mean values and standard deviations of the uncertain factors are listed in **Table 1**.

Table 1 Uncertain factors of VRS model

Type	Vars	Unit	Mean	St D
Steel S235 mechanical properties	*RY*	MPa	284.5	21.5
	RM	GPa	203	12.6
	SY	MPa	284.5	21.5
	SM	GPa	203	12.6
	PY	MPa	284.5	21.5
	PM	GPa	203	12.6
Tolerances of fabrication	*RT*	mm	3	0.15
	ST	mm	3	0.15
	PT	mm	5	0.25
Installation uncertainties	*SoilM*	MPa	400	100
	BP	N	12432	4144

Performances of the VRS were evaluated by the two criteria [3]:

- Theoretical Head Impact Velocity (*THIV*): as the vehicle changes its speed during contact with the safety feature, the head of the occupant continues moving freely until it strikes an inner surface of the vehicle with the velocity *THIV*;
- The Working width (*W*) is the distance between the traffic face of the restraint system and the maximum dynamic lateral position of any major part of the system. Dynamic Deflection (*Dd*) is the maximum lateral dynamic displacement of the side facing the traffic of the restraint system (see **Fig. 6**). *Dd* is used to measure the deformation of VRS during the impact process.

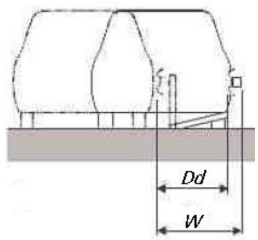

Fig. 6 Measurement of the VRS deformation

The SA are summarized in **Table 2**: Two-level screening with Orthogonal Array (OA) and multi-level screening with Morris analysis (MA) were used to identify the influential uncertain factors and Sobol' indices was used to quantify the influences of the influential ones. 2 out of the 11 uncertain factors (*PY*, *PT*) were identified as of great influence on VRS performances and their influences were quantified. Instead of considering the uncertainties of all the 11 uncertain factors, only the uncertainties of *PY* and *PT* will be studied in the MONO.

Table 2 SA of the VRS

Eleven uncertain factors	***RY, RM, SY, SM, PY, PM, RT, ST, PT, SoilM, BP***
Step1: OA screening	with 12 model runs
Factors chosen after OA	***RY, RM, PY, RT, ST, PT***
Step2: MA screening	with 42 model runs
Factors chosen after MA	***PY, RT, PT***
Step3: Sobol' indices	with 120 model runs to create the surrogate model
Critical factors	***PY, PT***

3.3 Objects and Constraints of VRS optimization

The objectives of optimization are to minimize *THIV* and *Dd*, with model *Mass* (i.e. price of installation) as constraint. Both formula (4), (6) will be used for the definition of objectives and constraints in MONO and the deterministic optimization results will also be calculated as comparison. Model *Mass* uncertainties are caused by tolerances of *PT*, and the maximum deviation of *Mass* remains nearly the same. The influence of uncertain factors on *Mass* is neglected.

3.4 Design variables of VRS

VRS components are illustrated in **Fig. 7**. The dimension parameters *H, E, A, B* are used as design variables. The under boundary and upper boundary of each design variable is pre-defined as initial value decrease and increase by 20%.

50 samples are taken through LHS and the performances of VRS are analyzed in the pre-defined design space: The decrease of rail dimensions, especially *E*, degrades the redirection capability of the VRS (see **Fig. 8**); Decrease of post dimensions increases greatly the deflection of VRS. The design intervals are re-defined and are listed in **Table 3**.

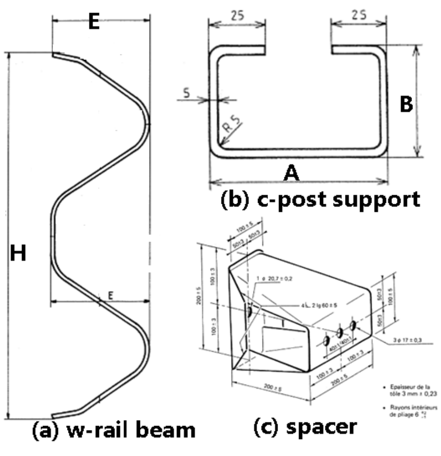

Fig. 7 The components of VRS

Fig. 8 Failure of vehicle redirection when *H* and *E* decrease

Table 3 Intervals of design variables

Variables	*Under*	*Initial/mm*	*Upper*
H	-15%	310	+25%
E	-10%	81	+25%
A	-20%	100	+25%
B	-15%	50	+25%

3.5 Creation of VRS surrogate model

240 samples are taken with LHS in the inputs space of 6 dimensions (4 design variables and 2 uncertain factors). The scatter plots of uncertain factors *PT*, *PY* and model outputs *THIV*, *Dd* are illustrated in **Fig. 9**. Kriging interpolation is used to create the surrogate mode, with *H, E, A, B, PT, PY* as inputs and *THIV*, *Dd, Mass* as outputs.

3.6 Optimization of VRS

3.6.1 Simplification of Objectives

It is evident that *THIV* has positive correlation with uncertain factors, and *Dd* is the opposite (see **Fig. 9**). The objects of the optimization problem is to minimize *THIV* and *Dd*, and we are interest in the maximum value of a design causing by uncertain factors (i.e. $\max_{\mathbf{p}} \mathbf{F}(\mathbf{x},\mathbf{p})$ in function (4)(6)). $\max_{PT,PY} THIV$ is obtained when *PT*, *PY* take their maximum values and $\max_{PT,PY} Dd$ is obtained when *PT*, *PY* take their minimum values. Assuming *PT*, *PY* have normal distributions and their mean values and standard deviations are shown in eq.(7), the maximum and minimum values of *PT*, *PY* are taken by eq. (8), with their Cumulative Distribution Function (CDF) values shown in eq.(9):

$$\begin{aligned} PT(MPa) &\sim N(284.5, 21.5) \\ PY(mm) &\sim N(5, 0.25) \end{aligned} \tag{7}$$

$$\begin{cases} \min PT = 4.68mm \\ \max PT = 5.32mm \end{cases} \& \begin{cases} \min PY = 256.95MPa \\ \max PY = 312.05MPa \end{cases} \tag{8}$$

$$\begin{cases} CDF(\min PT) = 0.1 \\ CDF(\max PT) = 0.9 \end{cases} \& \begin{cases} CDF(\min PY) = 0.1 \\ CDF(\max PY) = 0.9 \end{cases} \tag{9}$$

Monte Carlo designs are used to test the rationality of eq. (8): For different designs $\mathbf{x}_i$, LHS is used to test the relationship between uncertain factors and outputs with the surrogate model. With min*PT*, max*PT*, min*PY*, max*PY* defined in eq. (8), we have:

$$\begin{aligned} &\Pr\left(THIV(\mathbf{x}_i) \le THIV(\mathbf{x}_i, [\max PT, \max PY])\right) > 97.5\% \\ &\Pr\left(Dd(\mathbf{x}_i) \le Dd(\mathbf{x}_i, [\min PT, \min PY])\right) > 97.5\% \end{aligned} \tag{10}$$

The Outer-Inner optimization problem defined in eq. (4)(6) requires $n_{out} \times n_{in}$ model runs (n_{out}: number of model runs for outer minimization; n_{in}: number of model runs for inner maximization). The Inner maximization process is simplified, which greatly reduced model runs of the optimization problem; In addition, the surrogate model is only need to be validated in the design space, with the uncertain factors *PT*, *PY* fixed at specified values.

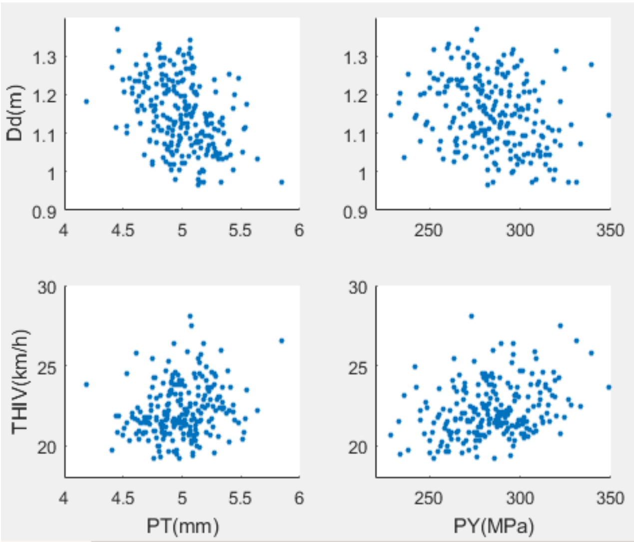

Fig. 9 Scatterplot of *PT*, *PY* and outputs *THIV*, *Dd*

3.6.2 Define of constraints

In order to define *Mass* constraint and limits of outputs deviations $\mathbf{\Delta_F}$ (see eq. (6)), 1000 samples are generated with LHS for uncertainty study of model outputs: model *Mass* varies in the interval [610 810]kg; deviation of *THIV* (i.e. max$THIV(\mathbf{x}_i,\mathbf{p})$-$THIV(\mathbf{x}_i,\mathbf{p}_0)$) varies in the interval [0.17 1.84]km/h; deviation of *Dd* (i.e. max$Dd(\mathbf{x}_i,\mathbf{p})$-$Dd(\mathbf{x}_i,\mathbf{p}_0)$) varies in the interval [37 102]mm. The constraints are determined artificially according to the uncertainties of model outputs as:

$$\begin{aligned} &Mass \le 730kg \\ &\max THIV\left(\mathbf{x}_i,\mathbf{p}\right) - THIV\left(\mathbf{x}_i,\mathbf{p}_0\right) \le 1km/h \\ &\max Dd\left(\mathbf{x}_i,\mathbf{p}\right) - Dd\left(\mathbf{x}_i,\mathbf{p}_0\right) \le 70mm \end{aligned} \quad (11)$$

3.6.3 Results analysis

Matlab is used to create the surrogate model and the automate design and optimization platform Isight is used for multi-objective optimization with Genetic Algorithm. Precision of the surrogate model is validated with the proposed method. With eq. (11) as constraint, the Pareto efficient solutions are illustrated in **Fig. 10**. Designs in Region 1 reduced both *Dd* and *THIV* , and designs in Region 2 are preferred when the main objective of the optimization is to decrease deformations of the VRS during the crash process. Scatterplots in **Fig. 11-14** illustrate the relationship between inputs' scaling factors and the outputs of the Pareto efficient designs. The optimal solution can be chosen depending on the requires of designer and the 5 designs *a*, *b*, *c*, *d*, *e* (see **Fig. 10**) are studied:

- For all optimization solutions, inputs *E*, *B* are proposed to be increased with their scaling values change in interval [1.16 1.24] and interval [1.16 1.25] respectively;
- The value of input *A* is proposed to be decreased (design *b*, *c*, *d*, *e*) when the minimization of *Dd* isn't of critical importance; *A* is proposed to be increased and *H* is proposed to be decreased in situations where the main object of the optimization design is to minimize deformation and to increase containment level of the device.
- From solutions *a* to *e*, the dimensions of the VRS support Post (i.e. inputs *A*, *B*) tend to decrease, and the scaling factor of input *H* need to be increased properly in order to maintain the optimal state.
- The under design limit for the scaling factor of input *A* is 0.8 and the upper design limit for the scaling factor of input *B* is 1.25 in this study. Unfortunately, these two limits restrained the selection of Pareto efficient designs (see **Fig. 13**, **Fig. 14**). Better solutions might be found beyond these two limits.
- In addition, more materials are needed in order to increase the rigidity of the VRS and decrease the output *Dd*, the constraint *Mass* defined in the optimization problem mainly restraint the minimization of *Dd*.

In short, the dimensions of the *w*-beam Rail component, especially for input *E*, need to be increased in order to increase the energy absorption capability of the VRS. The Post component is of rectangular shape (see **Fig. 7**) with *A*=100mm, *B*=50mm.The input *A* is proposed to be decreased and input *B* is proposed to be increased in the optimization process. **Table 4** listed the criteria of the optimized design *e* and the initial design. The optimization could improve both the performances of the VRS and its robustness.

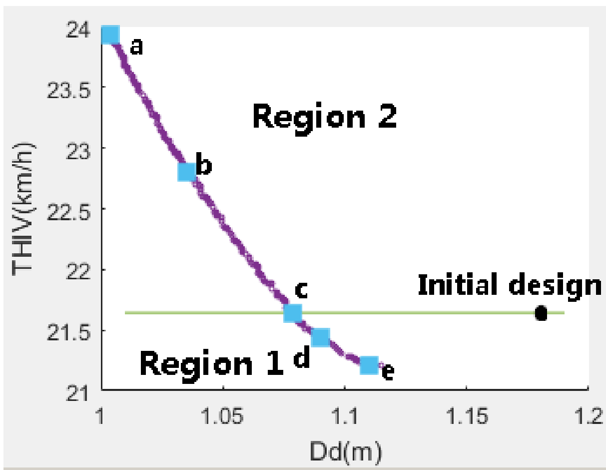

Fig. 10 Pareto efficient solutions of VRS MONO

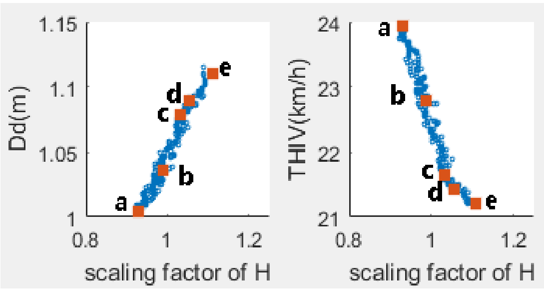

Fig. 11 Scatterplot of *H* and outputs of Pareto efficient solutions

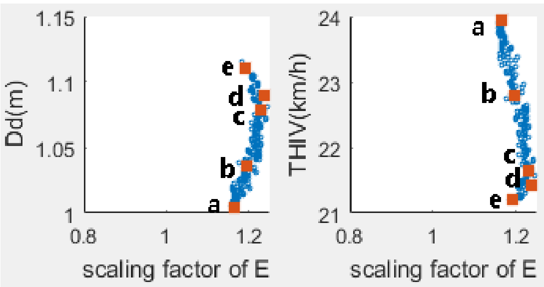

Fig. 12 Scatterplot of *E* and outputs of Pareto efficient solutions

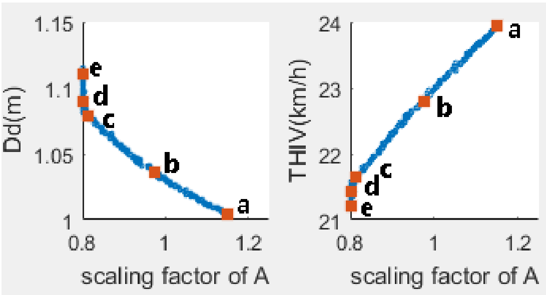

Fig. 13 Scatterplot of *A* and outputs of Pareto efficient solutions

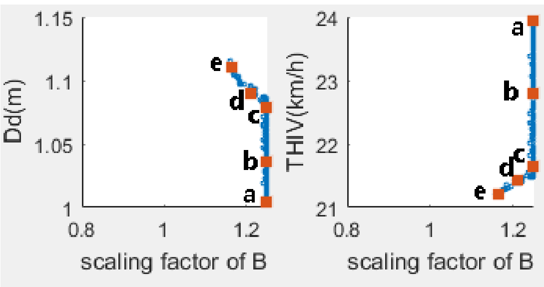

Fig. 14 Scatterplot of *B* and outputs of Pareto efficient solutions

Table 4 Performance criteria of initial and the optimized design

	Severity	*Def*	*Robust criteria*	
	THIV (km/h)	*Dd* (m)	*ΔTHIV*	*ΔDd*
Initial	21.66	1.180	1.09	0.072
e	21.19	1.111	0.46	0.070

3.6.4 Comparison of different MONO methods

Fig. 15 compared the optimal solutions of MONO (eq. (4) as objectives) with deterministic optimization. **Fig. 16** compared the optimal solutions of MONO (eq. (6) as objects) with deterministic optimization. For the optimization of VRS, we have:

- In **Fig. 15**: Outputs values and their possible maximum values obtained with MONO coincide with those of deterministic solutions. And this MONO method hasn't increase evidently the model robustness relative to deterministic designs.
- In **Fig. 16**: The influences of uncertain factors on the performance criteria of the VRS, especially *Dd*, decrease with the increase of model rigidity. The optimal solutions with low model deformation *Dd* and relatively high rigidity are selected with the robust method. Robust Optimization with eq. (6) select the optimal designs with outputs uncertainties within the limit Δ_F. The value of Δ_F influences the robustness of the model and the selection of optimal designs.

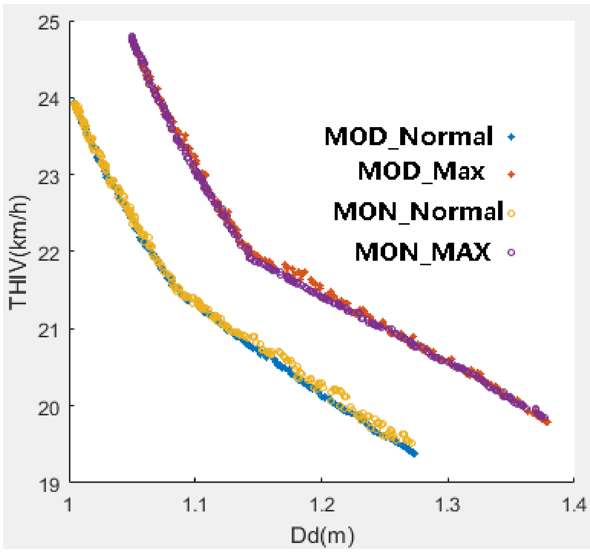

Fig. 15 Optimal designs obtained with Multi-Objective Deterministic (MOD) and with Non-deterministic (MON)

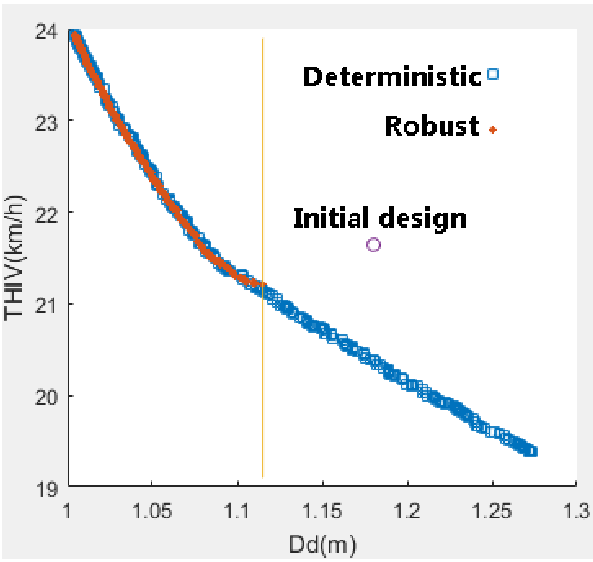

Fig. 16 Optimal designs of VRS obtained with different methods

3.7 Generalization of impact conditions

The performances of the VRS are optimized under the specified crash conditions. In fact, the real crash accidents are more complex:

- The installation conditions of road equipment are innumerable. Straight longitudinal barriers are tested although curved installations exist. Flat ground is recommended even though installations are situated sometimes on sloped shoulders or behind curbs;
- The errant vehicle may of various types (bus, truck, car, even motorcycle). Crash speed & angle, crash position, friction coefficient of road surface and the tire, etc. are not fixed factors.

The optimized design *e* (see **Fig. 10**) is evaluated under generalized test conditions. Restrained by numerical model, only the crash velocity and angle are considered. Simulations of the design with velocity (*v*) and angle (*a*) at different levels are realized and relationship between crash conditions and performance criteria of the VRS are studied:

- With polynomial regression analysis: relationships between *a* and *v* with output *THIV* at values (18 21 24 27 30) km/h are created and shown in **Fig. 17**. The relationship functions are listed in eq.(12); relationships between *a* and *v* with working width (*W*) [3] at levels (*W*2 *W*3 *W*4 *W*5 *W*6) are created and shown in **Fig. 18**. The relationship functions are listed in eq.(13).
- The VRS are fail to redirect the vehicle only at the extreme crash conditions, e.g *v*=130km/h, *a*>32° or *v*>100km/h, *a*=32°. The threshold (fail line) under which the device has well redirect the vehicle is shown with dotted line in **Fig. 17** and **Fig. 18**;
- In all possible crash conditions, the accident severity is of level **A** [3] with *THIV*<33km/h. The accident severity are restrained at acceptable level;
- The device works well for small value of impact angle: we have *THIV*<18km/h and *W*<*W*3 when *a*=10°, even for *v*=130km/h.
- Increment of *a* will greatly increase the severity of accident and the deformations of the VRS.

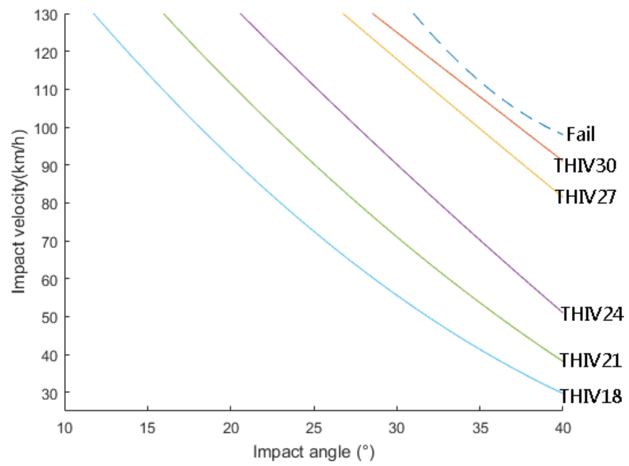

Fig. 17 Relationship between *a* and *v* with *THIV* at different levels

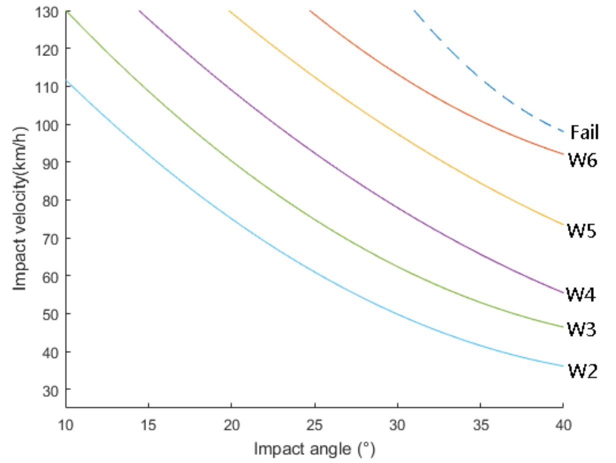

Fig. 18 Relationship between *a* and *v* with *W* at different levels

$$
\begin{aligned}
v_{THIV18} &= 0.0525a^2 - 6.2697a + 196.4188 \\
v_{THIV21} &= 0.0379a^2 - 5.9393a + 215.1071 \\
v_{THIV24} &= 0.0158a^2 - 5.0282a + 226.7711 \\
v_{THIV27} &= 0.0052a^2 - 4.0126a + 233.6191 \\
v_{THIV30} &= -3.4a + 227
\end{aligned} \quad (12)
$$

$$
\begin{aligned}
v_{W2} &= 0.0576a^2 - 5.4024a + 160.0000 \\
v_{W3} &= 0.0598a^2 - 5.7790a + 181.9330 \\
v_{W4} &= 0.0429a^2 - 5.2486a + 196.8000 \\
v_{W5} &= 0.0400a^2 - 5.2000a + 217.4000 \\
v_{W6} &= 0.0700a^2 - 7.0100a + 260.4500
\end{aligned} \quad (13)
$$

4 Conclusions

The development of VRS is a complex process. In Europe, the norm EN1317 specified the crash conditions under which a VRS must be tested before being installed along the roadside and defined the performance criteria of a device. However, the crash conditions are numerous, uncertain factors may degrade the performances of a design, and various design factors need to be considered in the optimization process. The procedure for MONO of complex engineering systems is studied and a VRS is optimized with the proposed process:

- Before the optimization process, numerical model need to be simplified to reduce single model run cost. And influential factors of which the uncertainties should be considered in optimization process need to be identified with SA;
- The MONO minimizes the outputs with their deviations constrained in limited intervals. The design space and model outputs uncertainties need to be evaluated before the MONO design;

- Surrogate model are used to substitute the high calculation cost model in optimization problems. Accuracy of a surrogate model needs to be ensured in order to secure the precision of optimization. Instead of evaluate the surrogate model across the whole inputs space, refinement of samples around the potential optimal solutions could greatly reduce the additional samples required to create an accurate surrogate model;
- Constraints and objectives can be of different forms depending on the demands of designers. The VRS is optimized with robust method, and strategies are proposed for optimization of the device.
- Optimal solutions obtained with different methods are compared: the robust method with eq. (6) as objectives is preferred for the MONO of the VRS.
- The optimized design *e* shown in **Fig. 10** is chosen. Performances of the design are evaluated under different crash conditions and the relationships between the impact speed and the impact angle are created with the performance criteria defined at different levels. The optimized device is capable to redirect the errant vehicles at almost all the crash conditions and to restrain the accident severity at level **A**. And it fails to redirect the errant vehicles only at extreme crash conditions.

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