Numerical and experimental elbow flow evaluation of dense fluids at high Reynolds number

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Abstract

The turbulent flow of dense fluids through piping singularities such as section changes or bends may be the source of steady-state low-frequency vibrations, which can lead to fatigue failure. The appropriate modeling of the unsteady flow through a singularity is an important step in the evaluation of flow-structure interaction phenomenon. In this study, numerical simulations of water flowing through a 90° circular bend at Reynolds number $5.6 \times 10^5$ were compared to experimental data in order to validate the computational approach. Particle Image Velocimetry (PIV) techniques were employed to experimentally characterize the water flow at multiple planes upstream, downstream and over a transparent bend. Large-Eddy Simulations (LES) of the water flow for the same Reynolds number and elbow geometry were conducted and made it possible to compare the
numerical and experimental velocity profiles in the bend exit and downstream of it. The average and fluctuating velocity fields obtained by the simulations are validated and a discussion of the main flow coherent structures formed in the bend is proposed.

1. Introduction

Industrial piping systems, originally designed for static charges, are subjected to dynamic loadings that may lead to significant vibration levels, which in turn can possibly result in fatigue failure. One of the loading sources responsible for piping vibration is the turbulent flow of dense fluids within the system itself. If the internal flow of liquids in industrial applications is generally turbulent, the fluid-structure interaction phenomenon is much more significant around piping singularities such as sudden changes of the cross-section area or of the flow direction. At these spots, the flow is disturbed and becomes highly turbulent, provoking a broadband low-frequency excitation that might possess enough energy to result in wide-range periodic displacement of thin-walled structures.

The understanding of this dynamic behavior is currently of great importance to the energy sector, among others, that seeks to be able to model the flow-induced vibration phenomenon within their piping systems. The correct modeling of the flow through a piping singularity is therefore of utmost importance in this process, as its unsteady pressure field constitutes the excitation mechanism.
The present study focuses on the 90° bend singularity, whose flow configuration has historically interested many researchers. This is mostly due to a set of large-scale motions that form over the elbow and are transported downstream of it. The discovery of a secondary flow motion downstream of the 90° bend in the form of two “twin” counterrotating vortices sharing the cross section of the duct for low Reynolds flows is credited to Dean [1]. Although Boussinesq [2] had already found these vortices from the solution of the viscous and incompressible Navier-Stokes equations applied to a square-sectioned bend 8 years earlier, they are commonly known as Dean Vortices. Thunstall and Harvey [3] are the first to consider, for greater Reynolds numbers, the influence of the interaction of an eventual separation of the flow over the intrados of the elbow with the Dean Vortices. In these cases, it is observed that the vortices are no longer symmetric but acquire an oscillatory behavior, whereby one of them briefly seems to dominate the other alternately, in a motion more recently named Swirl-Switching.

Rutten et al. [4] employ Large-Eddy Simulations (LES) to evaluate typical Swirl-Switching frequencies and state that this alternate motion does not present a bi-stable configuration but is rather continuously oscillatory. Eguchi et al. [5] compare LES to Direct Numerical Simulation (DNS) of the pressure fluctuations induced by the elbow flow over the bend at multiple points on its walls. The authors validate LES as a tool for obtaining the unsteady pressure field around the elbow, while highlighting the importance of the input of correct initial conditions to the inlet flow, as velocity profile and turbulence distribution. Using a $k - \varepsilon$ turbulence model, Dutta and Nundi [6] analyzed the flow downstream of the
elbow for Reynolds numbers varying from 1 to $10 \times 10^5$ and concluded that the oscillating motion of the Dean Vortices presents almost no dependence on the upstream mean velocity for the studied range.

Particle Image Velocimetry (PIV) was also largely used to study the coherent motions found in elbow flow. Ono et al. [7] evaluate the influence of the bend curvature radius on the flow separation over the intrados using velocity fields obtained by PIV measurements on a seamless elbow cast in polyurethane resin. Time-resolved PIV allows Takamura et al. [8] to identify typical dimensionless frequencies for Dean Vortices oscillation. The authors also observe a typical separation frequency, since it oscillates back and forth over the intrados, and another typical frequency linked to the vortices shed by this separation region. It is also remarkable that these dimensionless typical frequencies do not strongly depend on the Reynolds number, the behavior being similar for certain flow velocity ranges [9]. More recently, Hellström et al. [10], Kalpakli and Orlü [11] and Vester et al. [12] apply Proper Orthogonal Decomposition (POD) to the velocity fields on cross-sections downstream of the elbow measured by PIV; the POD allows for the detailed study of the large-scale motions observed and how they are affected by the upstream flow conditions, confirming the hypothesis stated by Sakakibara and Machida [13]. Finally, Röhrig et al. [14] compared RANS and LES simulations to PIV data on cross-sections downstream of the elbow, validating the LES velocity and pressure fields calculations.

The present study compares LES and PIV data on longitudinal sections of the 90° bend, upstream and downstream of it as well. The average and fluctuating velocity
fields are used to validate the LES approach as a robust tool to assess the coherent flow structures that characterize elbow flow. The pertinence of the pressure field calculated over the whole domain using the numerical approach is also assessed and a discussion is proposed on whether the LES is an appropriate tool to provide unsteady data of the flow excitation of the structure for flow-induced vibration modeling.

2. Materials and methods

2.1. Experimental setup

In order to fully characterize the flow over a 90° elbow, a transparent elbow was placed in a closed water loop as shown in Figure 1. A centrifugal Grundfos pump ensures constant flow rate, which is verified by an electromagnetic Endress Hauser flow meter located upstream of the pump. Two tanks act as pressure dampeners and define the test zone limits. Within this zone, the ducts are made of transparent Unplasticised Polyvinylchloride (PVC-U), diameter $D_i = 0.0994$ m.

The flow exiting the tank number 1 goes through the grid located 10 diameters upstream of the elbow entrance and tagged with the number 3 in Figure 1. The grid is meant to break down the large flow structures issued by the upstream flow into homogenized turbulence, equally distributed over the duct section. Three different grids were used in order to evaluate the influence of the inlet flow turbulence length on the resulting topology of the elbow flow. The main difference between the grids is the mesh size, which breaks down inflow turbulence into smaller or larger eddies. The characteristics of each grid are
illustrated in Figure 2; the mesh is the finest on Grid 1 and grows on grids 2 and 3. The open area percentage is 73%, 79% and 85%, respectively.

The elbow (4) has a curvature radius \( r = 1.5D \) and is made of Polymethyl Methacrylate, also known as PMMA or acrylic glass. This material was chosen for its good optical properties, which are suitable for laser velocimetry techniques in fluid dynamics. The elbow is composed of two distinct blocks wherein the circular duct was cut, with a straight section approximately \( 1.5D \) long located upstream of the elbow entrance and another one \( 2.5D \) long downstream of its exit. The outer faces of the elbow are planes, in order to reduce refraction and consequently optical distortions on the particle images used for the PIV. The two halves were glued together, along the vertical curved joint plane, therefore avoiding a junction-type flaw on the symmetry plane of the bend. The resulting elbow is screwed to square flanges that connect it to the rest of the circuit. The flanges are also connected to a marble table with rigid supports, as is the grid holder (the supports are not illustrated in Figure 1 for the sake of visual clarity).

Two flow rates were studied, as shown in Table 1:

### 2.1.1. PIV 2D-2C

Particle Image Velocimetry on two-dimensional sections for the measurement of two velocity components (PIV 2D-2C) was applied to five different planes situated on the bend, upstream and downstream of it, as illustrated in Figure 3. The \( l \) coordinate follows the transparent elbow center-line from its most upstream
position \((l = -1.5D)\) to its furthest downstream section \((l = 2.5D)\), and is used in this study as the universal reference. Over the elbow, the universal coordinate assumes the form of an angle \(\theta\) going from 0 to 90°.

Two local coordinate systems are defined in order to facilitate the representation of the velocity fields on the different measurement planes: \(A = x, y, z\), located at \(l = -1.5D\) and \(A' = x', y', z'\), located at \(l = 0\). Their positions are shown on Figure 3.

A pulsed Yag laser of 190 mJ and a maximum frequency of 15 Hz is used as the light source for the PIV measurements. The flow is seeded with polyamide particles with diameters ranging from 32 to 50 μm; the polyamide/water solution has a concentration of approximately 5 g·m\(^{-3}\). The particle images are taken with a LaVision\textsuperscript{R} camera with a 2048 x 2048 pixel CCD sensor. In order to obtain the desired image width, a 60 mm lens is used. Finally, an optical filter is added to the lens in order to block out all wavelengths different from that of the Yag laser, 532 nm. Figure 4 shows the examples of the measurement set-ups for sections 3 and 5. The laser and the image acquisition are synchronized using DaVis, provided by LaVision\textsuperscript{R}. For each measurement plane and flow configuration, 1000 pairs of particle images are taken at a frequency of 7 Hz; the timelapse within a pair of images is 300 μs for the flow configuration at 2 m.s\(^{-1}\) and 120 μs for the flow at 5 m.s\(^{-1}\). The images are first corrected to eliminate distortions and optical aberrations; this is done with a camera calibration model, developed using a reference image of a two-dimensional plate with multiple round marks on it, whose diameters and spacing are well known (please refer to David \textit{et al.} [15] and
Riethmuller et al. [16] for details on camera calibration techniques). The calibration model is a third order polynome that includes distortion correction.

The corrected images are then treated to filter out reflections from the laser sheet by extracting the average gray level of the image set. Finally, iterative cross-correlation was applied to the pairs of images in order to obtain the particle displacements; first, square windows containing 64 x 64 pixels are applied twice and then reduced to 32 x 32 pixel windows, with two passes, as well. 50% overlap between windows was applied.

2.2. Numerical approach

The software StarCCM+ (CD-Adapco®) was used for all the numerical simulations of the elbow flow mentioned in this study. The simulated flow configuration is that of water at $Re = 5.6 \times 10^5$ with respect to the duct’s inner diameter, which corresponds to a bulk velocity of 5 m.s$^{-1}$. The fluid domain corresponds to the geometry of the elbow used in the experiments, but with longer straight sections (2D long upstream of the elbow entrance and 4D long downstream of its exit). Two meshing techniques were employed to generate the volume mesh: first, prism layers originating at the walls and stretching towards the pipe core are used in order to allow for the correct boundary layer modeling. The resulting prism layer is 20 mm thick and is composed of 49 sublayers piled up in the wall’s normal direction. Since the sublayers stretch up with a 1.15 ratio, the first cell on the wall is 3.5 μm thick, which ensures a dimensionless wall distance $y^+ \leq 1$ at all solid boundaries. Secondly, a polyhedral mesh with a
targeted cell size of 5 mm is set for the pipe core. The final volume mesh counts 570 000 cells.

A Reynolds Averaged Navier-Stokes (RANS) simulation of the flow is first performed in order to initialize the turbulence over the elbow domain. A $\kappa$-$\varepsilon$ turbulence model with high $y^+$ wall treatment is used in an incompressible steady simulation. No-slip velocity condition is imposed on the domain’s walls and the outflow boundary is subjected to a pressure outlet condition. The inflow boundary condition corresponds to a velocity inlet condition, which is the focus of a more detailed treatment. As primarily evoked by Thunstall and Harvey [3] and then more recently discussed in detail by Eguchi et al. [5] and Sakakibara and Machida [13], the elbow flow features are highly dependent on the inlet flow conditions. Indeed, the upstream flow plays a major role in the separation and in the secondary flow that is formed on the intrados and convected downstream of the bend. In order to correctly simulate a completely developed turbulent duct flow, data obtained by the PIV measurements were injected in the inlet boundary of the RANS simulation.

The velocity profiles obtained at $l = -1.5D$ for each one of the grids were averaged and then an eighth degree polynomial function was fitted onto it, resulting in the inlet velocity configuration used in the RANS simulation. The second profile to be injected is that of the Turbulence Intensity. A similar procedure was followed to generate the turbulence intensity profile, which was calculated from the PIV measurements in its bi-dimensional form:
where the root mean square values of the velocity components are calculated using the velocity fluctuations over \( N = 1000 \) fields measured by PIV, each one at a different instant \( t_i \). Finally, the third quantity to be injected in the inlet boundary in order to model the upstream flow is the Turbulent Integral Length. This integral length had to be estimated from the PIV velocity fields, by integrating the correlation map obtained for the velocity fluctuations on the streamwise direction. With \( R_{u'}(x, x^*) \) as the correlation of the streamwise velocity fluctuations at a point \( x \) with relation to any other point of the measurement domain \( x^* \), we have:

\[
R_{u'}(x, x^*) = \langle u'(x, t_i), u'(x^*, t_i) \rangle, \quad i = 1, ..., N
\]

Let \( s \) be the streamwise coordinate of the position \( x \) and \( r \) its normal coordinate. The turbulent integral length at a distance \( r \) from the duct wall is calculated as the integral of the correlations \( R_{u'}(\{r, s_0\}, \{r, s\}) \) where \( s_0 \) is the reference point and \( s \) starts at \( s_0 \) and ends where the correlation reaches zero. Nevertheless, since the PIV measurement planes are limited in the streamwise direction to a \( 1.5D \) length, the correlations do not reach zero within the velocity field’s limits. Hence, what is calculated is an estimate of the Turbulent Integral Length, \( L_{u'}^\ast(r) \), where the integral boundary is limited to the available maximum \( s \), as follows:

\[
L_{u'}^\ast(r) = \int_{s_0}^{s_{\text{max}}} R_{u'}(\{r, s_0\}, \{r, s\}) ds
\]
After convergence of the RANS solution, turbulence is considered sufficiently initialized over the fluid domain and the LES calculation can then take place. The inlet flow conditions are the same as those used for the RANS simulation. The Synthetic Eddy Method proposed by Jarrin \textit{et al.} [17] is used to generate an unsteady turbulent channel flow possessing the turbulent characteristics specified under the form of the Turbulence Intensity and Turbulent Integral Length scale profiles obtained by PIV (Figure 5). The Implicit Unsteady solver is employed with a time-step of $5 \times 10^{-4}$ to ensure a Current Flow Number inferior to the unit, and a second-order temporal discretization. Smagorinsky is used as the Subgrid-Scale Model in the present LES formulation. Velocity and pressure data are taken over multiple sections of the volume mesh, in the form of snapshots analogous to the velocity field realizations of the PIV measurements. The sampling frequency for data extraction is 100 Hz and the solution duration is 20 seconds, resulting in 2000 velocity snapshots on each of the five reference sections illustrated in Fig 3.

3. Results

Figure 6 illustrates the average velocity field measured with the PIV technique using 1000 snapshots taken on sections 1 to 5; grid 2 was used and the bulk velocity $U_0$ is 5 m.s$^{-1}$. It can be seen on Section 1 that the flow accelerates in the inner part of the elbow even before the bend starts. This accelerated jet is intensified over the bend, leading to flow separation. For the configuration
illustrated here, the flow separates on the intrados at the position $\theta = \theta_2 = 51.5^\circ$. Depending on the grid and bulk velocity, this position can range between $50^\circ$ and $53.5^\circ$, as shown in Table 2. The finer the grid mesh, the smaller the homogenous turbulent inflow structures, which results in the separation taking place further downstream on the intrados. The higher bulk velocities, on the contrary, tend to lead to earlier separation. None of the studied configurations presented a clear reattachment point downstream of the elbow exit, which is in accordance with measurements by Ono et al. [7].

In order to compare numerical results of the elbow flow to the PIV measurements, this study focuses on the sections located downstream of the elbow. Figure 7 illustrates the LES results and multiple configurations of the PIV measurements on Section 4; it pictures the axial and transversal average velocity profiles as well as the turbulent intensity profiles at five different positions downstream of the elbow exit. The velocity and turbulent intensity profiles obtained by PIV present little difference according to the mesh size of the grid used to break down the inflow turbulence. The PIV velocity profiles for bulk velocity of 2 and 5 m.s$^{-1}$ superpose very well, indicating that the flow dynamics do not change significantly within this Reynolds number range. Nevertheless, the $\tilde{v}/U_0$ profiles, mainly at positions $x' = 0$ and $x' = 0.5D$ clearly show that the separation takes place earlier for the 5 m.s$^{-1}$ case compared to the 2 m.s$^{-1}$ one. The axial and transversal velocity profiles present a couple of inflexion points, first located close to the elbow intrados and then moving upward in the $y'$ direction as positions further downstream are observed. This indicates the presence of a mixing layer
originating at the intrados, growing larger as the flow moves downstream. The mixing layer is also evidenced by the turbulent intensity profiles; the intensity is higher close to the inner part of the elbow, as clearly seen at position $x' = 0$, where the flow is separated and exits the bend. As the flow goes further downstream, the intensity peak moves upward, towards the elbow extrados.

The axial velocity profiles ($\bar{u}/U_0$) at cross-sections $x' = 0$ and $x' = 0.5D$ can be compared to PIV measurements performed by Ono et al. [7]. The authors measured the velocity profiles for a similar Reynolds number ($5.4 \times 10^5$) and same curvature radius ($\gamma = 1.5D$), but for a different duct diameter $D_i = 150 \text{ mm}$. The reference profile at $x' = 0$ does not present the multiple inflexion points that exist in all the flow configurations measured in this study for the same cross-section. Instead, the profile is rather monotonic and it is only in cross-section $x' = 0.5D$ that the inflexion appears. This result indicates that, even if the ratio $\gamma/D$ is kept constant, different pipe diameters may lead to differences in the flow separation.

The LES profiles capture the separated zone and correct velocity and turbulent intensity levels, but seem to anticipate the separation point and therefore the mixing layer growth. This anticipation is clearly seen in Figure 8 where the magnitude of velocity on section 4, calculated by the LES, is compared to the topology given by the PIV measurements. The experimental data shows the mixing region, separating an elongated low velocity zone located beneath it from a high velocity zone above it. While thin at the elbow exit ($x' = 0$), it gradually
grows to occupy approximately half of the duct’s diameter at the cross-section located at $x' = 2D$. The LES results picture a larger mixing region, indicating the precipitated prediction of the separation point; the numerical mixing layer is located a little above the duct’s centerline by the cross-section $x' = 2D$. Figure 9 shows the velocity and turbulent intensity profiles at different cross-sections of Section 5. The LES sensibly overestimates the $\bar{u}/U_0$ profile at the domain entrance and then anticipates the low-velocity region located around the pipe centerline. This low velocity region is a consequence of the mixing layer growing larger and reaching Section 5. While the profiles on Section 4 indicate that the separation takes place earlier for the 5 m.s$^{-1}$ configuration, axial velocity and turbulent intensity profiles measured at cross-section $x' = 1D$ for Section 5 show that the mixing layer develops faster for the low velocity case. The $\bar{w}/U_0$ profiles picture the presence of the Dean Vortices all across the domain of Section 5. This can be seen clearly in Figure 10, which compares the flow’s topology for the transversal component (here it is $\bar{w}$) obtained by LES with that obtained by PIV using Grid 3 and bulk velocity $U_0 = 5$ m.s$^{-1}$. LES correctly predicts the presence of the Dean Vortices, their size and their spreading across the entirety of Section 5.

4. Conclusion

A Large-Eddy Simulation of highly-turbulent elbow flow was performed; in parallel, Particle Image Velocimetry was used to obtain experimental data for
multiple configurations of water flow through a transparent 90° elbow designed for the purpose of this study. Multiple planar sections were studied allowing for a restitution of the tridimensional flow over the entire domain. The most important coherent flow motions were identified in the experimental and numerical data.

Little influence of the inflow turbulence configuration, which was controlled using different grid meshes upstream of the elbow, could be noticed. The region close to the separation point is an exception, for the influence of the grid mesh and the bulk velocity can be perceived on the velocity field’s transversal component. Indeed, separation takes place earlier for the high velocity case but the separation region grows faster for the low velocity one. The important features of the flow remained practically unchanged within the studied Reynolds number range. The numerical simulation could capture the main coherent flow structures and features, although the position of the separation point was anticipated and the growth of the mixing layer was slightly overestimated.

5. Acknowledgements

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References


complex turbulent flow in a short elbow piping under high Reynolds number


Figure legends

Figure 1 – Left: schematic view of the water loop; right: exploded view of the transparent elbow

Figure 2 – Dimensions (in mm) of the grids used for turbulence control

Figure 3 – Coordinate system and reference sections studied by PIV

Figure 4 – Schematic views of the PIV set-ups for Section 3 (left) and Section 5 (right)

Figure 5 – Turbulence intensity and estimated length scale profiles applied on the inflow boundary

Figure 6 – Average velocity field obtained by PIV; Grid 1, U₀ = 5 m.s⁻¹

Figure 7 – Velocity and turbulent intensity profiles for multiple cross-sections of Section 4

Figure 8 - Velocity magnitude topology on Section 4. Top: PIV (Grid 3, U₀ = 5 m.s⁻¹); Bottom: LES

Figure 9 – Velocity and turbulent intensity profiles for multiple cross-sections of Section 5

Figure 10 – Transversal component’s (\(\bar{w}/U₀\)) topology on Section 5. Top: PIV (Grid 3, U₀ = 5 m.s⁻¹); Bottom: LES
Table legends

**Table 1** – Flow configurations experimentally studied

**Table 2** – Separation point position for each of the flow configurations

**Tables**

1-

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<thead>
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2-

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