OPTIMAL DESIGN OF VERTICAL SLOTS FISH LADDER

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Abstract. A fish ladder (or fishway, fish pass) is a hydraulic structure constructed near dams. Such fish passage permits to immigrant fishes to cross to theirs productions and feeding areas or from the cold area to the hot one for species which do not withstand the cold period.
Our purpose is to establish an optimal structure that allows fish to pass through the dams with less effort. In order to achieve this challenge, we will give a mathematical formulation of channel composed of ten pools with vertical slots for obtaining a flow pattern effective for a wide range of species.
We proceed with the study of the state system given by the shallow water equations and the objective function which is related to fish's swimming aptitudes. Numerical simulations for ten pools channel are given to illustrate the efficiency and viability of the technique.

Key Words: Shallow water equations, optimal shape design, fishways, hydraulic engineering.

1. Introduction

Many species of salmon, shad, giant catfishes, dourado, sturgeons and eel migrate between the sea and the rivers to complete their life cycle. Free migration routes for fish are crucial to their survival. We take interest in diadromous fish species which immigrate between sale and fresh water.

We distinguish some types of diadromous fish: Anadromous fish (as salmon, smelt, American shad, hickory shad, striped bass, lamprey, gulf sturgeon,...) which live and grow in the salt water, and migrate to freshwater rivers and lakes to reproduce. The Anadromous fish eventually return to freshwater to spawn. About half of all diadromous fish in the world are anadromous. Adult Catadromous (American eel, European eel, inanga, shortfin eel, longfin eel) live in fresh water, then migrate to the sea for breeding. After hatching, they migrate back to freshwater where they stay until growing into adults. Catadromous fish undertake a great migration from freshwater to spawn in the marine, and they die there due to the effort made for migration. About one quarter of all diadromous fish in the world are catadromous. Amphidromous species (bigmouth sleeper, mountain mullet, sirajo goby, river goby, torrentfish, Dolly Varden) migrate between estuaries and coastal rivers and streams (in both directions). Amphidromous fish live in freshwater for breeding and they leave to the marine for feeding and growing.

The presence of dams without fish passes appears to be a major contributing factor in the decline of migratory species. Fishways have been designed to provide safe passage for migratory species inhabiting the river to get pass towards their breeding or feeding areas. The utility of such systems has been demonstrated around the world. The best general reference here is Clay [3] for the pool and weir type, Katopodis et al. [4] for the Denil fishways and for the vertical slot type Rajaratnam et al. [8].

Vertical slot ladders are quite common and use a large narrow slot to control water flow and depths in the pools between slots. This allows fish to swim upstream without leaping over an obstacle. This design reasonably handles the seasonal fluctuation in water levels and is not sensitive to impoundment or upstream water surface elevation changes.

The paper is devoted to the study of vertical slots fishway. The aim of this work is to assess the possibility of using a two-dimensional shallow water model to compute the flow pattern in vertical slot fish ladder and deduce an optimal structure allows fish to cross the obstacle in a convenient conditions.

The reminder of the paper is organized as follows. The next section is dedicated to the mathematical formulation and the introduction of the objective function related to the optimal shape design. Then, in the third section, a finite volume discretization for the shallow water equations using a total variation diminishing (TVD) scheme combined with a gradient free algorithm is proposed. The last section provides numerical results obtained for a standard ten pools channel.

2. Mathematical model

A vertical-slot fishway, shown in Figure 1, is a rectangular channel $\Omega \subset \mathbb{R}^2$ with a sloping floor that is divided into 10 pools by baffles. The pools have a double function: they ensure a proper dissipation of the energy of water flowing through the fish pass, and provide resting areas for the fish. It is worth pointing out that the geometric features of each pool are with a width of $0.97m$, a length of $1.213m$, also two transition pools, one at the beginning and other at the end of the channel with the same width and a length of $1.5m$. Inside each pool, two baffles are built. They have a width of $2r = 0.061m$ and are vertical to the lateral fishway boundary. The channel is constructed with a slope relative to the ground.

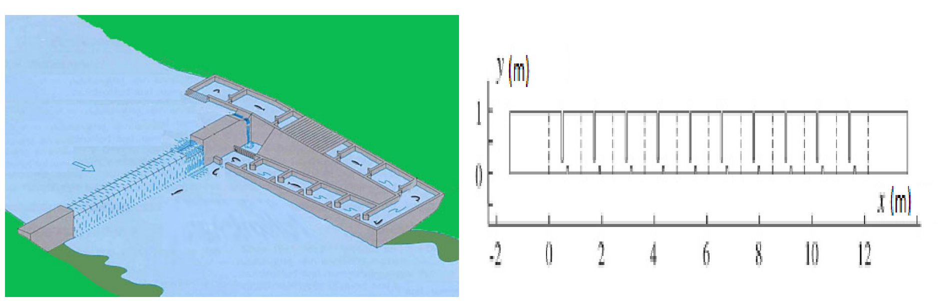

Figure 1. Fishway and ground plan Ω: each pool is designed by dashed lines

The shallow water equations are used to simulate a variety of problems related to environment and coastal engineering. These equations can be obtained by integrating the incompressible Navier-Stokes equations in depth and taking into account the kinematic and kinetic boundary conditions. The 2-D shallow-water equations with source terms may be written as

$$\begin{aligned} &\frac{\partial H}{\partial t} + \overrightarrow{\nabla}.\overrightarrow{Q} = 0 && \text{in } \Omega \times (0,T) \\ &\frac{\partial \overrightarrow{Q}}{\partial t} + \overrightarrow{\nabla}.(\overrightarrow{Q} \otimes \frac{\overrightarrow{Q}}{H}) + gH\overrightarrow{\nabla}(H-\eta) = \overrightarrow{f} && \text{in } \Omega \times (0,T) \end{aligned} \tag{2.1}$$

Where H is the water depth; $\overrightarrow{u} = (u_1, u_2)$ is the velocity vector; u_1 and u_2 are the x and y components of flow velocity, respectively; $Q = (u_1 H, u_2 H)$ is the unit-width discharge; η is the bottom geometry; g is the gravitational acceleration and $\overrightarrow{f}$ represents all effects of bottom friction and atmospheric pressure.

We introduce three parts of the boundary of Ω: the lateral boundary denoted by γ_0, the inflow boundary denoted by γ_1, and the outflow boundary denoted by γ_2. We take for $\overrightarrow{n}$ the unit outer normal vector to boundary. To obtain a well-posed problem, we add to this system an initial and boundary conditions defined by:

$$\begin{aligned} &H(0) = H_0, \quad \overrightarrow{Q}(0) = \overrightarrow{Q}_0 \text{ in } \Omega, \quad \overrightarrow{Q}.\overrightarrow{n} = 0 \qquad curl\left(\frac{\overrightarrow{Q}}{H}\right) = 0 \text{ on } \gamma_0 \times (0,T), \\ &\overrightarrow{Q} = Q_1 \overrightarrow{n} \text{ on } \gamma_1 \times (0,T), \qquad H = H_2 \text{ on } \gamma_2 \times (0,T) \end{aligned} \tag{2.2}$$

The geometry of the vertical slot based on the use of guide elements to lead smooth hydraulic flow into the next slot. The positioning of the guide elements was carried out at two different locations, a and b, which configure the shape of the fish ladder Ω (Figure 2).

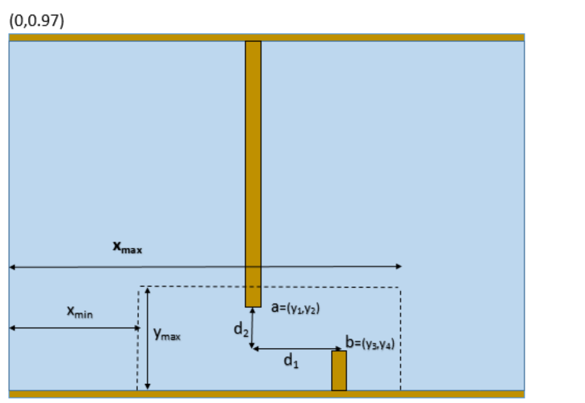

FIGURE 2. Prototype geometry: details of slot and pool

The design variables a and b are subject to constraints in order to ensure a positive influence on the flow in the individual pools. These constraints are formulated as

$$\begin{aligned} &\frac{1}{4}\,1.213 \leq y_1, y_3 \leq \frac{3}{4}\,1.213 \\ &0 \leq y_2, y_4 \leq \frac{1}{4}\,0.97 \end{aligned} \tag{2.3}$$

In order to provide a comfort conditions during the fish passage and permit to a maximum number of fishes to pass to the river upstream, the following constraints are introduced

$$\begin{aligned} &y_3 - y_1 \geq d_1 = 0.1 \\ &y_2 - y_4 \geq d_2 = 0.05 \end{aligned} \tag{2.4}$$

A shape optimization problem consists in the minimization of a functional $J \in \mathbf{R}$, also called cost function depending on the design variables a and b defining the shape within the admissible constants defining the admissible set X. We have a direct calculation loop for the functional: from a parameterization $(a,\ b)$ we define a domain $\Omega(a,\ b)$ on which we compute the state equation solution $W = (H,\ Q)$ and the cost function $J(\Omega(a,\ b))$:

$$J : x = (a,\ b) \in X \longrightarrow \Omega(x) \longrightarrow W(\Omega(x)) \longrightarrow J(x, \Omega(x), W(\Omega(x)))$$

We consider that the shape of the structure is efficient if the associated energy dissipation leads to a velocity of water close to a target velocity $\vec{v}$ related to fishes species and minimizing the flow turbulence in the channel.
The target velocity is given by

$$\overrightarrow{v}(x_1, x_2) = \begin{cases} (c, 0) & \text{if } \ x_2 \leq \frac{1}{3}\,0.97 \\ (0, 0) & \text{otherwise} \end{cases} \tag{2.5}$$

Thus, we want to minimize the following cost function

$$J = \frac{1}{2}\int_0^T \int_\Omega \| \overrightarrow{u} - \overrightarrow{v} \|^2 \, dxdt + \frac{\alpha}{2}\int_0^T \int_\Omega |\ rot(\overrightarrow{u})\ |^2 \, dxdt \tag{2.6}$$

with $\alpha \geq 0$ is the vorticity parameter, and $\overrightarrow{u} = \dfrac{\overrightarrow{Q}}{H}$ where $(H, \overrightarrow{Q})$ is solution of the shallow water system (2.1) with the initial and boundary conditions (2.2).

3. Numerical resolution

The shallow water equations are a set of nonlinear hyperbolic equations. The nonlinear character combined with the hyperbolic type of the equations can lead to discontinuous solutions in finite time. In order to formulate simple and robust numerical procedures, the two-dimensional shallow water equations (2.1) are cast in conservation form with source terms

$$\partial_t U + \partial_x F(U) + \partial_y G(U) = S(U) \tag{3.1}$$

where

$$U = \begin{pmatrix} H \\ Hu_1 \\ Hu_2 \end{pmatrix},\ F(U) = \begin{pmatrix} Hu_1 \\ Hu_1^2 + \frac{1}{2}gH^2 \\ Hu_1u_2 \end{pmatrix},\ G(U) = \begin{pmatrix} Hu_2 \\ Hu_1u_2 \\ Hu_2^2 + \frac{1}{2}gH^2 \end{pmatrix}$$

and

$$S(U) = \begin{pmatrix} 0 \\ f_1 + gH\partial_x\eta \\ f_2 + gH\partial_y\eta \end{pmatrix}$$

3.1. Finite volume method.

Finite volume schemes for the shallow water systems consist in using an upwinding of the fluxes. The problem domain is first discretized into a set of triangular cells T_i forming an unstructured computational mesh. Let Δt be the constant time step and define $t_n = n\Delta t$ for $n = 0, ..., N$. At each discrete time t_n, we note Q_i^n the approximated flux value and H_i^n the approximated height value.

Denote by $E(U) = (F(U), G(U))$ the physical fluxes. By integrating the equation (3.1) on a triangle T_i, we obtain

$$\int_{T_i} U_t + \int_{T_i} \nabla.E(U) = \int_{T_i} S(U) \tag{3.2}$$

We note $\vec{n_i}$ the normal on the edges of triangle T_i. Using the divergence formula $\int_{T_i} \nabla.E(U) = \int_{\partial T_i} E(U).n_i d\Gamma$. The equation (3.2) takes the form

$$\int_{T_i} U_t + \int_{\partial T_i} E(U).n_i d\Gamma = \int_{T_i} S(U) \tag{3.3}$$

The term $\int_{\partial T_i} E(U).n_i d\Gamma$ can be calculated as

$$\int_{\partial T_i} E(U).n_i d\Gamma = \sum_{j=1}^{3} E_{ij}.n_{ij}.dl_{ij}$$

The equation (3.3) becomes

$$| T_i | U_t + \sum_{j=1}^{3} E_{ij}.n_{ij}.dl_{ij} = | T_i | S \tag{3.4}$$

where n_{ij} is the normal on the edge T_i/T_j, E_{ij} are the discrete fluxes on the interface

T_i/T_j and dl_{ij} is the length of the interface T_i/T_j.
Thus, we now have an equation for each cell i of the form

$$U_t = -\frac{1}{\mid T_i \mid}\sum_{j=1}^{3} E_{ij}.n_{ij}.dl_{ij} + S \tag{3.5}$$

We make a finite difference approximation to the time derivative to obtain the scheme

$$U^{n+1} = U^n - \frac{dt}{\mid T_i \mid}.\sum_{j=1}^{3} E_{ij}.n_{ij}.dl_{ij} + dt.S \tag{3.6}$$

Finding the value of the fluxes at the interface is of primary importance. A variety of approximation techniques have been developed to allow efficient calculation of the solution to the Riemann problem. The Roe solver is used to evaluate the term $\sum_{j=1}^{3} E_{ij}.n_{ij}.dl_{ij}$.

3.2. **A gradient free algorithm.**

The design variables related to the shape Ω depend on the two positions of the slot $y = (a,\ b) = (y_1,\ y_2,\ y_3,\ y_4)$ (Figure 2). We redefine the objective function (2.6) in the following way $\Phi_1 : \mathbb{R}^4 \to \mathbb{R}$ where $\Phi_1(y) = J(\Omega(y))$. The finite volume scheme (3.6) yields, for each time t_n, an approximated velocity $\overrightarrow{u}_i^n = \dfrac{\overrightarrow{Q_i^n}}{H_i^n}$ which induces an approximate objective function

$$\bar{\Phi}_1(y) = \frac{\Delta t}{2}\sum_{n=1}^{N}\sum_{e\in T_i}[\int_e \parallel \overrightarrow{u}_i^n - \overrightarrow{v} \parallel^2 + \alpha \int_e \mid curl(\overrightarrow{u}_i^n) \mid^2] \tag{3.7}$$

Finally, we collect all linear constraints (2.3) and (2.4) in a function $\overrightarrow{\phi}_2$: $\mathbb{R}^4 \to \mathbb{R}^{10}$

$$\begin{aligned}\overrightarrow{\phi}_2(y_1, y_2, y_3, y_4) = \quad & (\frac{1}{4}1.213 - y1,\ \frac{1}{4}1.213 - y_3,\ y_1 - \frac{3}{4}1.21,\ y_3 - \frac{3}{4}1.213,\ -y_2, \\ & -y_4,\ y_2 - \frac{1}{2}0.97,\ y_4 - \frac{1}{2}0.97,\ 0.1 - y_3 + y_1,\ 0.05 - y_2 + y_4)\end{aligned} \tag{3.8}$$

These constraints are satisfied if and only if $\overrightarrow{\phi}_2(y) \leq 0$. The functional $\bar{\Phi}_1$ can be penalized to include geometric and state constraints

$$\Phi(y) = \bar{\Phi}_1(y) + \beta \sum_{j=1}^{10} max\{(\overrightarrow{\phi}_2(y))_j, 0\} \tag{3.9}$$

with β is a penalty parameter.

Due to the essentially geometric nature of the problem, we propose a direct search technique for solving the discretized control problem. The NelderMead “simplex” algorithm is one of the most widely used methods for nonlinear optimization. The method attempts to minimize a scalar-valued nonlinear function using only function values, without any derivative information. The method constructs a sequence of simplices as approximations to an optimal point. To describe Nelder-Mead iterations, we begin with an arbitrary simplex of 5 vertices $y_1, y_2, ..., y_5$. We evaluate and order our function on these vertices $\Phi(y_1) \leq \Phi(y_2) \leq ... \leq \Phi(y_5)$. The vertex associated to the maximal value is replaced with a new point $y(\nu) = (1+\nu)y^* - \nu y_5$, where y^* is the centroid of the convex hull $\{y_1, ..., y_4\}$. The value of ν is chosen from this set of values: $\nu_\delta = -0.5$, $\nu_\gamma = 0.5$, $\nu_\alpha = 1$, $\nu_\beta = 2$. The choice of these values is determined according to the following algorithm

Calculate and sort $\Phi(y_1), \Phi(y_2), ..., \Phi(y_5)$
While $\mid \Phi(y_5) - \Phi(y_1) \mid$ is not sufficiently small, calculate $y(\nu_\beta)$ and $\Phi_\beta = \Phi(y(\nu_\beta))$ then

a) If $\Phi_\beta \leq \Phi(y_1)$ then calculate $\Phi_\alpha = \Phi(y(\nu_\alpha))$. If $\Phi_\beta \leq \Phi_\alpha$, replace y_5 with $y(\nu_\alpha)$; otherwise replace y_5 with $y(\nu_\beta)$. Go to (f)

b) If $\Phi(y_1) \leq \Phi_\beta \leq \Phi(y_4)$ then replace y_5 with y_β and go to (f)

c) If $\Phi(y_4) \leq \Phi_\beta \leq \Phi(y_5)$, then calculate $\phi_\gamma = \Phi(y(\nu_\gamma))$. If $\Phi_\gamma \leq \Phi_\beta$ replace y_5 with $x(\nu_\gamma)$ and go to (f). Otherwise go to (e)

d) if $\Phi(y_5) \leq \Phi_\beta$ then calculate $\phi_\delta = \Phi(y(\nu_\delta))$. If $\Phi_\delta \leq \Phi_{y_5}$, replace y_5 with $y(\nu_\delta)$ and go to (f). Otherwise go to (e)

e) For $j = 2, ..., 5$, set $y_j = y_1 + \frac{1}{2}(y_j - y_1)$

f) Resort values of Φ at each resulting vertex

To prevent stagnation at non-optimal point, a modification proposed by Kelley [5] is used. This technique consists to replace the current simplex by a smaller one.

3.3. **Numerical results.**

Several numerical simulations for three pool geometries with different configurations of baffles piers were conducted. All computations have been initialized with a constant initial and boundary conditions. In particular, $H_0 == 0.5m$, $Q_0 = (0;0)m^2s^{-1}$, $Q_1 = -0.065/0.97\ m^2s^{-1}$, and $H_2 = 0.5$ m. For the cost function, the target velocity is $c = 0.8\ m.s^{-1}$ with the vorticity parameter equals to $\alpha = 0$ and the penalty parameter $\beta = 500$. For the second member $\overrightarrow{f}$, only the bottom friction stress is carried out associated to Chezy coefficient of 57.36. The focus of the present study is to deal with a fish passage associated to a comfortable conditions, the geometric characteristics of the pool were $d_1 = 0.1$ and $d_2 = 0.05$.

3.3.1. *Experiment 1.*

For uniform flow conditions the flow pattern in vertical slot fishway depends mainly on the specific pool design. The fishway under study is shown in Figure 2.
Only results related to the central pool are shown. For the initial random shape Figure 3, we observe the recirculated flow near from the long baffle and the small

one compared to the optimal shape Figure 3, the curved trajectory of velocity is removed, and the velocity is close to the uniform target velocity v. The recirculation region near from baffles is removed for the optimal points $a = (0.5721, 0.1487)$ and $b = (0.8786, 0.0520)$.

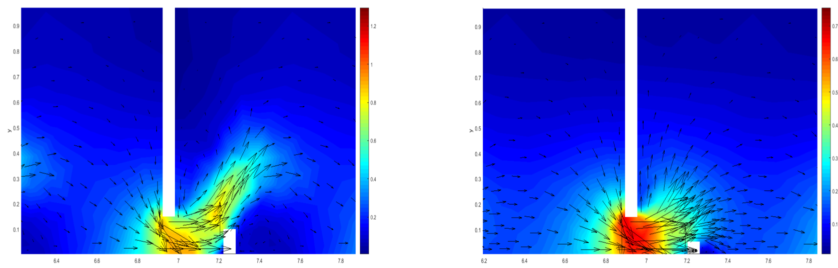

FIGURE 3. Initial (left) and optimal (right) velocities for central pool

3.3.2. *Experiment 2.*

The second test consists in a similar form of fishway but the vertical baffles are replaced by oblique baffles Figure 4.

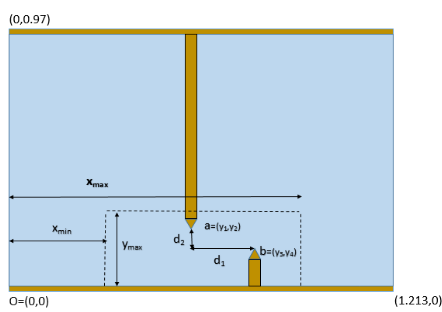

FIGURE 4. Scheme of the first pool

The results show a large recirculation region between the crosswalls for the random initial shape Figure 5.

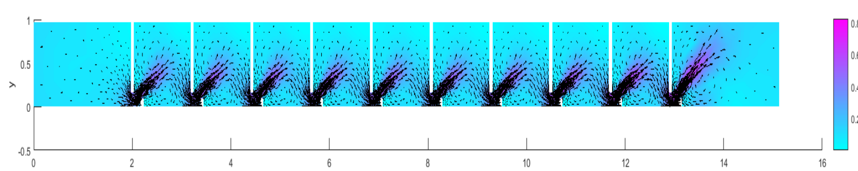

FIGURE 5. Non Optimal Shape and corresponding velocity

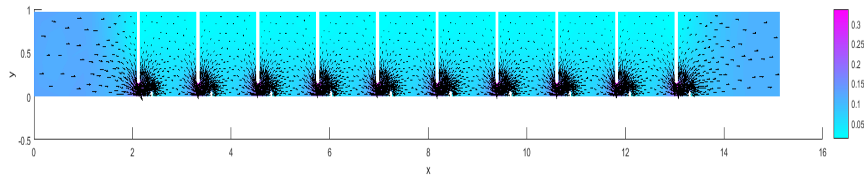

FIGURE 6. Optimal Shape and corresponding velocity

The optimal configuration shows a small swiriling zone in the slot region and far away, the velocity is close to the uniform target velocity Figure 6. The optimal points positions are $a = (0.6170, 0.1477)$ and $b = (0.8792, 0.0554)$.

3.3.3. *Experiment 3.*

The third test starts from a same rectangular form of fishway shown in Figure 1, but the two vertical baffles are replaced by three vertical baffles Figure 7.

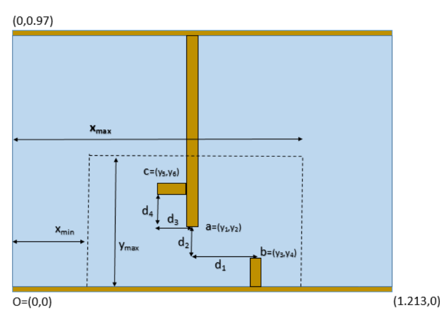

FIGURE 7. Scheme of the first pool

We consider the form of fishway depends now on the three points $a(y_1, y_2)$, $b(y_3, y_4)$ and $c(y_5, y_6)$. The comfort constraints are rewritten as

$$\begin{aligned} \frac{1}{4}\,1.213 \leq y_1, y_3, y_5 \leq \frac{3}{4}\,1.213 \\ 0 \leq y_2, y_4, y_6 \leq \frac{1}{2}\,0.97 \end{aligned} \tag{3.10}$$

The stability constraints take the form

$$\begin{aligned} y_3 - y_1 \geq d_1 = 0.1 \\ y_2 - y_4 \geq d_2 = 0.05 \\ y_1 - y_5 \geq d_3 = \frac{1}{2}0.0305 \\ y_6 - y_2 \geq d_4 = \frac{1}{2}0.0305 \end{aligned} \tag{3.11}$$

Finally, the objective function conserves the same writing. We collect all sixteen constraints defined in (3.10) and (3.11) in a function $\overrightarrow{\Phi}_2 : \mathbb{R}^6 \longrightarrow \mathbb{R}^{16}$

$$\overrightarrow{\phi}_2(y_1,y_2,y_3,y_4,y_5,y_6) = \begin{array}{l} (\frac{1}{4}1.213 - y1,\ \frac{1}{4}1.213 - y_3,\ \frac{1}{4}1.213 - y_5,\ y_1 - \frac{3}{4}1.21, \\ y_3 - \frac{3}{4}1.213\ y_5 - \frac{3}{4}1.213, -y_2,\ -y_4,\ -y_6,\ y_2 - \frac{1}{2}0.97, \\ y_4 - \frac{1}{2}0.97,\ y_6 - \frac{1}{2}0.97,\ 0.1 - y_3 + y_1,\ 0.05 - y_2 + y_4, \\ \frac{1}{2}0.0305 - y_1 + y_5,\ \frac{1}{2}0.0305 + y_2 - y_6) \end{array} \tag{3.12}$$

The associated penalty function takes the form

$$\Phi(y) = \bar{\Phi}_1(y) + \beta \sum_{j=1}^{16} max\{(\vec{\phi}_2(y))_j, 0\} \tag{3.13}$$

For numerical simulations, the geometric parameters are $d_1 = 0.1$, $d_2 = 0.05$, $d_3 = \frac{1}{2}0.0305$, $d_4 = \frac{1}{2}0.0305$. The obtained optimal position points are
$a = (0.5521, 0.1581)$, $b = (0.7770, 0.0818)$ and $c = (0.3500, 0.2431)$.

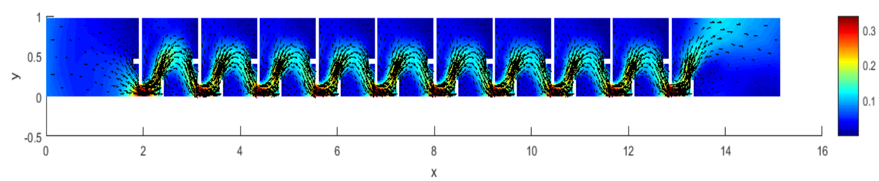

FIGURE 8. Non Optimal Shape and corresponding velocity

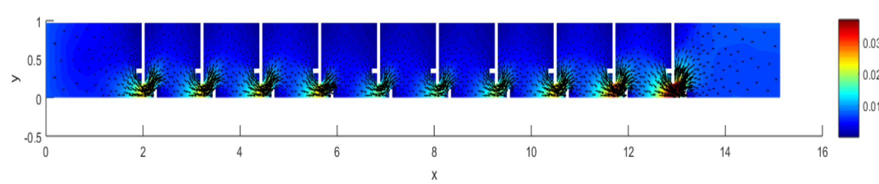

FIGURE 9. Optimal Shape and corresponding velocity

We observe, in the optimal configuration, that the circulation areas disappear improving the structure's hydraulic performance.

4. CONCLUSION

Simulations of variant configurations provide a detailed flow patterns in vertical slot fish ladder and allow to identify hydraulic issues and propose an appropriate type of construction.

The optimal shape design techniques combined with a robust total variation diminishing scheme for solving the state system can be considered as useful tools for practical fishway design purpose.

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